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MULTIWAVELENGTH LASER PROPAGATION STUDY

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J. Richard Kerr

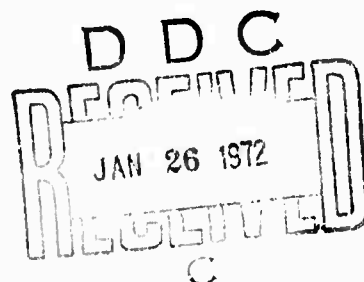
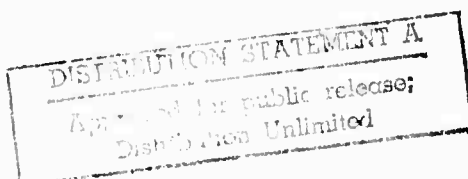
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13. ABSTRACT During the reporting period, preliminary experiments were conducted on the nature and effects of fundamental intermittencies in atmospheric turbulence. These intermittencies affect scintillation levels, statistics, and experimental data spread, to a much greater degree than has been generally recognized. Following this, attention was given to transmitter aperture effects, and current experiments are pointing out serious deficiencies in certain theoretical predictions. As an example, the concept of a focused beam seems largely meaningless in turbulence, and predictions of sharp reductions in scintillations under such a condition are not borne out by photographic and electronic observations. Finally, a recent series of comprehensive multiwavelength scintillation experiments was incorporated into a paper, with the addition of new interpretative material.		

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## SUMMARY

The purpose of this program is to experimentally investigate multiwavelength laser beam scintillation phenomena over horizontal paths, and to relate these effects to the characteristics of atmospheric turbulence. Field experiments are being conducted with the use of specialized instrumentation which was developed on the program. This instrumentation includes simultaneous and coincident multiple beams ranging from visible to middle-infrared wavelengths, with a very large receiver-dynamic-range and real-time processing of a variety of scintillation statistics. The transmitter and receiver configurations are variable from virtual-point to large apertures. The turbulence strength and structure is determined from microthermal measurements.

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condition are not borne out by photographic and electronic measurements. Finally, a recent series of comprehensive multiwavelength scintillation experiments was incorporated into a paper for publication, with new interpretative material.

The results of these efforts are applicable to target-illumination problems; to proposed transmitter diversity systems for alleviating such problems; and to receiver diversity approaches for image enhancement and improved performance of optical/infrared radar, reconnaissance, and communications systems.

## CONTENTS

	Page
I. Introduction	1
II. Turbulence Intermittency Effects	1
III. Transmitter Aperture Effects	2
IV. Future Work	3
Appendix A: Experiments on Turbulence Characteristics and Multiwavelength Scintillation Phenomena	A-1
Appendix B: Intermittency of Turbulence in Atmospheric Scintillation Phenomena	B-1
Appendix C: Transmitter Size and Focus Effects on Scintillations	C-1



## I. Introduction

During the period covered in this report, work was done on both turbulence intermittency and on transmitter aperture effects or scintillations. These will be described below. In addition, the series of comprehensive, multiwavelength scintillation and turbulence structure measurements which have been described in recent reports were incorporated into a paper for publication, along with new interpretative material.

The latter paper is included herein as Appendix A.

## II. Turbulence Intermittency Effects

The fundamental intermittency of turbulence affects scintillation levels, statistics, and experimental data spread to a much greater degree than has been generally recognized. A series of preliminary experiments was conducted in which such quantities as turbulence strength ( $C_n^2$ ) and scintillation level ( $\sigma^2$ ) were measured with relatively short averaging-times, and were in turn taken as related random variables. The implications are discussed in a short paper which is included as Appendix B, in which the need for more theoretical work is pointed out.

This topic will be the subject of more detailed efforts later in this program (Section IV).

### III. Transmitter Aperture Effects

For some time, there have existed analytical and intuitive arguments which predict a sharp reduction in scintillations for a receiver plane which is in the focused near field of the laser transmitter. The important implications for improved systems performance and target illumination are obvious, and "transmitter aperture averaging" has been utilized in a number of paper studies of proposed optical systems.

A series of experiments was conducted in order to demonstrate or disprove this effect. When the sharp reduction was not observed, attention was given to detailed numerical predictions which we obtained from a digital computer and complicated analytical expressions in the literature. These predictions pointed out factors which require more careful attention than was originally apparent. A review of the pertinent considerations is given in a short paper which is included as Appendix C, which also contains the applicable references.

It is now apparent that turbulence-induced beam spreading renders the concept of a focused beam rather meaningless, even in light turbulence. However, it is not immediately clear where the theoretical developments break down. Recent photographic work shows substantial beam break-up and a random replication of the transmitter diffraction scale at the receiver plane. The extreme criticality of focus as predicted by the theory is manifested by

a sharp decrease in beam break-up for a very narrow range of focus adjustments at the transmitter output. Turbulence-induced beam wander is also observed to play an important role, especially in signal fading at a fixed receiver.

We are presently conducting single-wavelength experiments involving photographic (qualitative) and electronic (quantitative) techniques. We are examining the properties of the received beam over a 1.4 km path, as a function of transmitter size and divergence (or convergence). These include the log amplitude variance and covariance, scintillation spectra, and probability distributions. In addition, we are utilizing variable receiver apertures from a virtual point up to 30 cm, which nominally intercepts the entire beam when focused. Data is being taken under both high and low turbulence conditions. As an important ancillary factor, we are examining the common assumption that a large, diverging source behaves like a point source as is often assumed in the scintillation literature.

In order to conduct these measurements, we have developed a spatial filtering mechanism which permits very precise control. The position of the 10 micron pinhole is resolvable to 2.5 microns in all three dimensions.

The results of the present work will be presented in the next report.

#### IV. Future Work

Following the completion of the transmitter aperture work,

we will move our field facility to a longer path of approximately 6 km, for operation during the summer and fall. This path will be near the ground, and will provide the largest path-integrated turbulence level of which we are aware.

The general types of multiwavelength measurements described in Appendix A will be repeated, and we expect to demonstrate the saturation of 10.6 micron scintillations for the first time. This should provide definitive answers to questions on the parameter-dependence of saturation, as well as enabling us to investigate the scintillation probability distributions, covariance curves, and receiver aperture smoothing for such an extreme case.

We will then return to a detailed study of intermittency effects.

## APPENDIX A

### **\*Experiments on Turbulence Characteristics and Multiwavelength Scintillation Phenomena**

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#### Abstract

Measurements of atmospheric turbulence structure and multiwavelength scintillation statistics are described. The scintillation measurements utilized coincident virtual point sources, and include log amplitude variances and covariances, spectra, and receiver aperture smoothing. These are related to turbulence strength, spectral slope, and inner scale.

The saturation of scintillations is found to be a wavelength-independent effect. The Kolmogorov atmospheric model breaks down under weak turbulence conditions, and hence the commonly used

atmospheric and propagation theories tend to apply under mutually contradictory conditions. The transverse amplitude correlation length and resultant receiver aperture smoothing depart from theoretical predictions under strong scintillations. Scintillation spectra show much data spread but averages support the Taylor hypothesis. Short-path optical determinations of turbulence strength are seriously affected by nonzero inner scales of turbulence. Correlations of multiwavelength scintillations vs. time indicate nonuniform turbulence spectra as well as strength over the path.

Further work is required on the effects of finite sources, and on the influence of turbulence intermittency on scintillation characteristics and data spread.

## I. Introduction

In this paper we describe a series of multiwavelength scintillation experiments with supporting measurements of the structure of atmospheric turbulence. The results have confirmed deficiencies in the commonly used atmospheric model and theoretical descriptions of propagation in a random medium.<sup>1</sup>

The experimental parameters are described in Table I, and the measurements are listed in Table II. The path was nominally uniform, and the three-wavelength measurements were made simultaneously and with spatially-coincident beams. Virtual-point-sources (spherical waves) were used in order to avoid ambiguities due to beam-wave effects.<sup>2</sup> Most of the data were processed in real time, utilizing highly-developed analog techniques. The microthermal and electro-optical instrumentation are described elsewhere.<sup>3</sup>

The results of turbulence structure measurements will be presented in the following section, followed by scintillation measurements in Section III. Further discussion is given in Section IV, and future experiments are described in Section V.

## II. Structure of turbulence

The structure of turbulence was measured through analysis of the temporal spectrum of microthermal fluctuations. In all cases, power-law behavior was evident, often with a breakpoint which with the mean wind speed defines an "inner scale." Substantial departure from the Kolmogorov or inertial subrange model<sup>1,4</sup> was often

noted.

In the single-scattering or Rytov realm,<sup>1</sup> any turbulence spectrum, once known, may in principle be utilized in an optical filter function (O.F.F.) analysis of the propagation problem.<sup>5,6</sup> However, as discussed below, there is clear evidence of nonuniformity under those conditions in which turbulence is not "highly-developed"--i.e., when substantial departure from the Kolmogorov spectrum is observed. It is therefore suggested that the related questions of turbulence nonuniformity, nonstationarity, and intermittence require extensive future consideration.<sup>7</sup>

The inner scale and the spectral slope as obtained from log-log plots are shown vs. the refractive index structure constant ( $C_n^2$ ) in Fig. 1. Since the analysis was done with an rms instrument, the theoretical or inertial subrange slope is (-0.833). The measured slopes are seen to depart substantially from this value, except at higher strengths of turbulence. The slopes tend to be smaller than the theoretical value, as has been observed by others (Section IV-B). Weak turbulence levels typically occur at night or with heavy cloud cover in the daytime. We have consistently observed anomalous conditions at sunrise and sunset,<sup>8</sup> but these results are omitted from the data presented here.

The slope and inner scale are given vs. wind speed in Fig. 2.



It is evident that anomalous spectra also tend to occur at wind speeds of 2 mph or less.

The customary definition for the turbulence strength parameter ( $C_n^2$ ) must be reconsidered for the case of a non-Kolmogorov turbulence. If  $z$  is the distance along the path and  $\kappa$  is the spatial wavenumber, it is usually assumed<sup>4</sup> that the overall spectral function  $\Phi_n(z, \kappa)$  is separable into a strength function  $C_n^2(z)$  and spectral function  $\Phi_{no}(\kappa)$ . However, as will be seen below, this is not always valid. Nevertheless, in predicting scintillation levels, we may usefully relate our temperature difference measurements between two appropriately-spaced points to the strength of turbulence and use the usual definition of  $C_n^2$ , implying a limited Kolmogorov spectrum as an approximation.

The departure from the Kolmogorov spectrum implies a scintillation dependence on wavelength which may depart substantially from the usual predictions. Furthermore, under strong turbulence conditions where the model is more realistic, it is well known that existing propagation analyses tend to break down due to the onset of multiple scattering or "saturation of scintillations".<sup>1</sup> Hence, it may be said that the two parts of the extant theory, i.e. the atmospheric model and the (Rytov) propagation analysis, have validity under mutually contradictory conditions.

Finally, it is well known that the inner scale may have a substantial effect on scintillations if it is large enough compared to the important spatial scale of the optical filter function.<sup>6</sup> We have observed highly-variable power-law behavior inside this scale. Unfortunately, this region of the spectrum may in many cases affect the scintillations considerably.<sup>9</sup>

### III. Scintillation measurements

#### A. Log amplitude fluctuations

The theoretical prediction for log amplitude variance of scintillations with a point source is<sup>1</sup>

$$\sigma_T^2 = 0.124 C_n^2 k^{7/6} L^{11/6}, \quad (1)$$

where  $k$  is the optical/IR wavenumber and  $L$  is the path length. The experimental values ( $\sigma_E^2$ ) are given in Figs. 3a-c for the three wavelengths respectively. Saturation of scintillations is evident at the two shorter wavelengths, with a fall-off beyond saturation ("supersaturation"). At  $10.6\mu$ , variances of more than 0.3 were observed with no evident saturation, indicating that saturation occurs at a scintillation level which is independent of wavelength. This agrees with dimensional considerations discussed in Sec. IV-A. The results for all three wavelengths are combined in Fig. 3d. The substantial experimental spread is discussed below and in Ref. 7.

In these figures, circles are used to indicate a "good inertial subrange" turbulence structure, arbitrarily defined as follows:

Spectral slope between -0.72 and -0.94

Inner scale corresponding to less than one-half the theoretical value of  $r_a$  (defined below)

In Fig. 4, we show the experimental variances for pairs of wavelengths. The lines represent the theoretical  $k^{7/6}$  dependence. Due to saturation, the points for high turbulence tend to a unity ratio, as has already been reported for near-IR and visible wavelengths.<sup>10</sup>

The scintillations were uniformly log normal regardless of wavelength or degree of saturation. The only exceptions were a few measurements made under very-low-wind conditions, where averaging (stationarity) was poor.

#### B. Short-path scintillation measurements

The short path system at 6328Å was utilized in an attempt to measure the "optical strength of turbulence", free from the effects of saturation. However, it was found that the log amplitude variance was consistently much smaller than that predicted by Eq. (1). This implies that  $\ell_o$ , the inner scale, is greater than  $\sqrt{\lambda L}$  for this path, and hence the appropriate theoretical expression is<sup>1</sup>

$$\sigma^2 = 0.32 C_n^2 L^{1/3} \ell_o^{-7/3} \quad . \quad (2)$$

We have utilized the thermally-determined value of  $C_n^2$ , with the measured log amplitude variance for this short path, to infer an inner scale from Eq. (2). In Fig. 5, these inner scale values are plotted against those obtained from the microthermal spectrum. Even though the theoretical basis for deriving Eq. (2) may be poorly satisfied in a real atmosphere, a scaled correlation is seen to exist.

Evidently a short-path "optical scintillometer" is of limited utility since the O.F.F. operates only on the high frequency portion of the spatial turbulence spectrum. Livingston has proposed a two-range method of optically determining the inner scale.<sup>11</sup>

#### C. Log amplitude covariance

We define the transverse log amplitude correlation length ( $r_a$ ) such that

$$\frac{C_\ell(r_a)}{C_\ell(0)} = \frac{1}{e}, \quad (3)$$

where  $C_\ell(r)$  is the log amplitude covariance function<sup>1,4</sup> and  $C_\ell(0) = \sigma^2$ . Experimental values of  $r_a$  are plotted in Fig. 6 vs. the strength of turbulence. The theoretical values are obtained from the analysis of Ref. 12.

It is seen that, at the shorter wavelengths, the values of  $r_a$  increase appreciably as the scintillations become saturated. This

agrees with certain results reviewed in Ref. 1, and implies poorer averaging of scintillations in a large receiver, as discussed below. Surprisingly, the values of  $r_a$  at  $10.6\mu$  are seen to decrease at higher turbulence levels.

Values of  $r_a$  at  $4880\text{\AA}$  are plotted vs. those at  $1.15\mu$  and  $10.6\mu$  in Figs. 7a,b respectively. The increase in  $r_a$  for the shorter wavelengths at high turbulence levels is quite evident.

The departure of  $r_a$  from theoretical values may be explained in terms of the failure of the atmospheric model at weaker turbulence levels, and of the propagation analyses at stronger levels. It may be mentioned that in the case of fog or rain, the observed values of  $r_a$  become small at all wavelengths.

#### D. Scintillation spectra

A theoretical treatment of scintillation spectra for the spherical wave case has recently been given.<sup>13</sup> If we define the spectral density of the log amplitude fluctuations<sup>14</sup> as  $W(f)$ , the frequency at which  $fW(f)$  is maximum is of interest. We denote this frequency as  $f_m$ , and note that the theory predicts the constancy of the dimensionless quantity<sup>1,4</sup>

$$\frac{f_m \sqrt{\lambda L}}{v_{\perp}} = (\text{constant}), \quad (4)$$

where  $v_{\perp}$  is the component of wind velocity perpendicular to the path. This is a consequence of the Taylor hypothesis, where  $\sqrt{\lambda L}$  represents

the "frozen-in" amplitude correlation length. In view of the covariance results presented above, it is suggested that  $\sqrt{\lambda} L$  be replaced by  $r_a$ .

Although our path was not sufficiently well instrumented to assure uniform wind conditions, it is possible to normalize-out the wind velocity. For example, in Fig. 8 we show the ratio of  $f_m$  at two wavelengths, vs. turbulence strength. The average of all points is within ten percent of the theoretical ratio implied by Eq. (4). It may be expected that the ratio will decrease under saturated conditions at the shorter wavelengths, due to the increase in  $r_a$ ; this has been observed elsewhere.<sup>1</sup> In the present case, no clear trend is evident, which suggests that the amplitude pattern may evolve more under high-scintillation conditions.

This conjecture is further supported by Fig. 9, in which the two-wavelength ratios of  $f_m r_a$  are plotted vs. turbulence strength. From the Taylor hypothesis, these ratios would be expected to be unity. Since the  $10.6\mu$  scintillations are not saturated, the increase which is observed at higher turbulence levels indicates increased evolution in the amplitude pattern at  $4880\text{\AA}$  and  $1.15\mu$ .

Scatter plots of  $f_m \sqrt{\lambda} L / v_{\perp}$  show a large spread, due probably to nonuniformity of the wind velocity over the path. The average of 48 points (all wavelengths) was 1.6. The quantity  $f_m r_a / v_{\perp}$  evidenced less spread and had an average value near unity. This compares favorably with the Russian results reported in Ref. 1.

#### E. Aperture averaging of scintillations ( $4880\text{\AA}$ )

The reduction in log amplitude variance for a 32 cm receiver relative to that for a virtual-point-receiver was less effective than predicted from theory,<sup>1,15</sup> or from a simple assumption of independently scintillating patches of size  $r_g$ . The ratio of log amplitude variances for large and small apertures is shown vs. turbulence strength in Fig. 10. Particularly poor aperture smoothing is evidenced under strongly saturated conditions, which has obvious implications for practical systems design.

The same ratio is shown in Fig. 11 vs. the size of  $r_g$ . As expected, an increase in  $r_g$  corresponds to poorer aperture averaging. This is more graphically illustrated in Fig. 12, where the abscissa is the value of the normalized covariance curve at a separation  $r=4$  cm.

Poor aperture smoothing has been attributed to transmitter aperture effects<sup>16</sup> and to nonhomogeneous paths.<sup>1</sup> We believe that neither is fundamental. The theoretical covariance curves for spherical and plane waves are similar,<sup>1</sup> and we believe that poor smoothing would also be observed for a plane wave source.

It appears that an adequate theoretical description of receiver smoothing remains to be given, even for non-saturated conditions. Equivalently, the covariance behavior must be known, including the details of the "tails" which have been observed to decrease more slowly than expected (Fig. 12 and Ref. 17). It would also be useful to extend the O.F.F. viewpoint<sup>5,6</sup> to the receiver smoothing case.

#### F. Nonstationarity of turbulence and scintillation levels

Under conditions of e.g. broken clouds, the level of scintillations and turbulence is seen to vary from moment to moment, and in fact the response to the sun emerging from a cloud is essentially instantaneous. The correlation of non-saturated log amplitude variances at three wavelengths, taken on such a day with 10 second averaging times, is shown in Fig. 13. A similar correlation is shown in Fig. 14, where thermal and short-path or "optical"  $C_n^2$  determinations were made at the receiver end, while the large cloud pattern drifted towards the transmitter and resulted in the time delays shown. In this figure, the scintillations at  $4880\text{\AA}$  are saturated, and evidence supersaturation, i.e. an anticorrelation with  $10.6\mu$  scintillation levels.

A more fundamental intermittency of turbulence exists in the absence of obvious causes such as broken-cloud skies.<sup>7</sup> This effect may be expected to result in nonuniform turbulence over a physically uniform path. The theoretical expression for the point-source log amplitude variance over a nonuniform path is<sup>1</sup>

$$\sigma^2 = 0.14 k^{7/6} \int_0^L C_n^2(z) (z/L)^{5/6} (L-z)^{5/6} dz. \quad (5)$$

From this expression, we see that wavelength does not appear in the path integral of  $C_n^2(z)$ , so that we would expect variances at the various wavelengths to remain correlated under varying conditions.



As shown in Fig. 15, this is not necessarily observed. One must conclude that results such as shown in this figure are caused by variable turbulence spectra over the path, and in fact on this particular day, a markedly non-inertial turbulence spectrum was measured.

#### IV. Further discussion

##### A. Saturation of log amplitude

The maximum saturated variance ( $\sigma_{\max}^2$ ) in Figs. 3 and 4 is approximately 0.6. This result, using an unambiguous point source, agrees with recent Russian measurements.<sup>1</sup> Many investigators have used larger, diverging sources. Under non-saturated conditions, this may be expected to give stronger scintillations than a point source, approaching a point source as the divergence is increased.<sup>2</sup> However, under saturated conditions, larger sources apparently give rise to lower maximum variances.<sup>1</sup> This may explain the lower ( $\sigma_{\max}^2 \approx 0.3$ ) variances observed in Ref. 8. The difference is quite important, since a rule of thumb for the signal dynamic range associated with a given log amplitude variance for log normal fluctuations is<sup>18</sup>

$$\text{Dynamic range} \approx 100 \sigma^2 . \quad (6)$$

We recommend against the use of the (linear) intensity or amplitude variance ( $\sigma_I^2$ ) in theoretical and experimental discussions. Although this quantity may be related to the log amplitude variance for an exactly log normal process,<sup>1</sup> it poorly characterizes such a

process physically. Furthermore, the conversion to a log variance for a real process is highly sensitive to skewness.<sup>19</sup>

In Figs. 3 and 4 we have presented evidence that the saturation curve for a point source is independent of wavelength out to  $10.6\mu$ . This is expected from dimensional considerations given below, and agrees with the results of Ref. 10 for visible and near-IR wavelengths.<sup>20</sup> There are some data showing an apparent wavelength-dependence,<sup>21</sup> but these may involve transmitter aperture and mode effects and constitute a small number of points at less than octave wavelength separations. From similar dimensional considerations, we expect no range-dependence of the saturation curve, although supersaturation may be incorrectly interpreted as implying a lower  $\sigma_{\max}^2$  at longer ranges. A range dependence can occur for distances small enough to involve inner scale effects.<sup>17</sup>

The wavelength- and distance-independence of saturation curves for a spherical (or plane) wave source in a Kolmogorov atmosphere may be predicted from the following dimensional reasonings. The only physical parameters are  $L$ ,  $\lambda$ , and  $C_n$ . For a given wavelength and strength of turbulence, we define a critical distance for saturation as

$$L_{\text{cr}} = \beta \frac{\lambda^m}{C_n^p} \quad , \quad (7)$$

such that

$$\begin{aligned}\sigma_{\max}^2 &= 0.124 C_n^2 k^{7/6} L_{\text{cr}}^{11/6} \\ &= 1.05 \beta^{11/6} \lambda^{(11m-7)/6} C_n^{(12-11p)/6} .\end{aligned}\quad (8)$$

The coefficient  $\beta$  must be dimensionless if we are to avoid introducing a scale length with no physical basis, and we expect it to be of order unity. Hence, we have

$$m + p/3 = 1 . \quad (9)$$

The choices ( $m=7/11$ ,  $p=12/11$ ) remove the three parameters from the saturation expression ( $\sigma_{\max}^2$ ), and result in  $\beta=0.73$  for  $\sigma_{\max}^2 = 0.6$ . A slightly different choice with simpler exponents is ( $m=2/3$ ,  $p=1$ ), giving  $\sigma_{\max}^2 \sim \lambda^{1/18} C_n^{1/6}$ . However, the value of  $\beta$  to fit experimental data is highly sensitive to this choice, and will now increase nearly an order of magnitude to allow for even this very weak wavelength-dependence. Since no other choices of  $m$  and  $p$  seem reasonable, we expect the fundamental parameter-independence of  $\sigma_{\max}^2$ . Of course, a finite transmitter aperture and/or significant inner scale will introduce scale lengths and modified parameter-dependencies.

It should be noted that the use of a gaussian (rather than Kolmogorov) atmosphere<sup>22</sup> introduces an artificial turbulence scale length, and leads to seriously erroneous parameter dependencies ( $\sigma^2 \sim Lk^2$ ) in the nonsaturated region. Although a number of authors have employed this model in saturation and other analyses, we believe

its use should be avoided.

Theoretical efforts to predict saturation and especially supersaturation have been largely unavailing, as reviewed in Ref. 1. Even in the nonsaturated region, the usual (Rytov) analysis has been attacked as being non-self-consistent,<sup>23,24</sup> non-energy-conserving unless corrected,<sup>25</sup> and as representing no real extension of the Born method.<sup>26-28</sup> However, data taken under highly-ideal conditions<sup>10</sup> show a small spread and confirm the basic theory.

Recent efforts<sup>29,30</sup> to describe saturation or multiple scattering are semi-empirical, involve a gaussian atmospheric model, and continue to be controversial.<sup>31,32</sup> A heuristic argument has been used to predict supersaturation.<sup>33</sup> Attempts at more rigorous solutions continue,<sup>34-36</sup> including a generalization of the Huygens-Fresnel method to a random medium,<sup>35,37</sup> and incorporation of the effects of turbulence intermittency.<sup>36</sup>

Young<sup>38-40</sup> has equated multiple scattering and "seeing" (wavefront distortion) effects, i.e. the smearing of the diffraction disc to beyond the  $\sqrt{\lambda} L$  scale. This agrees with our covariance observations in saturation. He describes saturation in terms of a virtual aperture-smoothing effect, and predicts the fall-off beyond saturation with increasing range (or turbulence) as  $L^{-q}$ . The value of  $q$  depends critically upon the MTF in the saturation region, and may be inferred from Ref. 40 and Coulman's MTF data<sup>41</sup> to be approximately 1/3. This viewpoint also predicts the wavelength-independence of  $\sigma_{\max}^2$ , and

saturation levels at radio frequencies are cited as evidence.<sup>40</sup>

#### B. Turbulence structure

The substantial data spread which is typical of scintillation experiments is believed to be due to the nonuniformity of turbulence, both in spectral form and in strength. The latter factor relates to the fundamental "intermittence" of turbulence, and is discussed in Ref. 7.

The degree of universality of the Kolmogorov spectrum is a matter of controversy. Under conditions of a nearly ideal terrain, constant wind, and extensive data averaging, the  $5/3$ -law spectrum has been verified under a wide variety of atmospheric conditions.<sup>42</sup> Further supporting data are given in Refs. 43-54. In fact, the  $5/3$  slope has been observed for scale sizes well beyond those of isotropy.<sup>51,55</sup> However, the present measurements, which show a tendency for a lesser slope, especially for weak turbulence, have also been well-supported elsewhere.<sup>17,56-58</sup> These latter results have been interpreted as evidencing an energy input in the subrange spatial scales. Further support for these departures from a simple atmosphere is given by qualitative interpretation of scintillation photographs.<sup>8</sup> The entire Kolmogorov theory is seriously questioned in Ref. 59, on the basis that large and small scale turbulence components are coupled. In any event, it is clear that for nonideal terrain, single measurements, and/or shorter averaging times, the inertial subrange atmospheric model is often inaccurate, and there is as yet no adequate theory.<sup>42</sup>

It is recognized that temporal spectra may not accurately represent true spatial spectra in the event of the breakdown of the "frozen-in" or Taylor hypothesis.<sup>1,4</sup> However, the trend toward smaller slopes agrees with that from direct spatial measurements.<sup>17,57</sup> Discussions supporting the Taylor hypothesis are given in Refs. 49 and 60. Limitations and modifications of the hypothesis are discussed in Refs. 51, 61, and 62; they do not appear important to the present discussion.

Finally, it should be remarked that the influence of random humidity variations on 10.6  $\mu$  scintillations remains to be adequately described.<sup>63</sup>

#### V. Future experiments

In current experiments we are examining the effects of transmitter size and divergence in detail.<sup>2</sup> Following this, multiwavelength long-path horizontal experiments will be conducted in order to demonstrate the saturation of scintillations at 10.6  $\mu$ ; to further verify the independence of the saturation curve with  $(\lambda, L)$ ; and to examine the behavior of  $r_a$ , probability distribution, and receiver aperture smoothing for very large path-integrated turbulence. Detailed attention will then be given to the intermittency problem.<sup>7</sup>

Further work is also needed in transmitter and receiver diversity systems for overcoming the effects of scintillations,<sup>64</sup> and in

relating MTF measurements to beam wander, spread, and scintillations.<sup>37,65</sup> Finally, there is a great need for extensive measurements of the characteristics of turbulence vs. altitude and their effects on vertically propagating beams.

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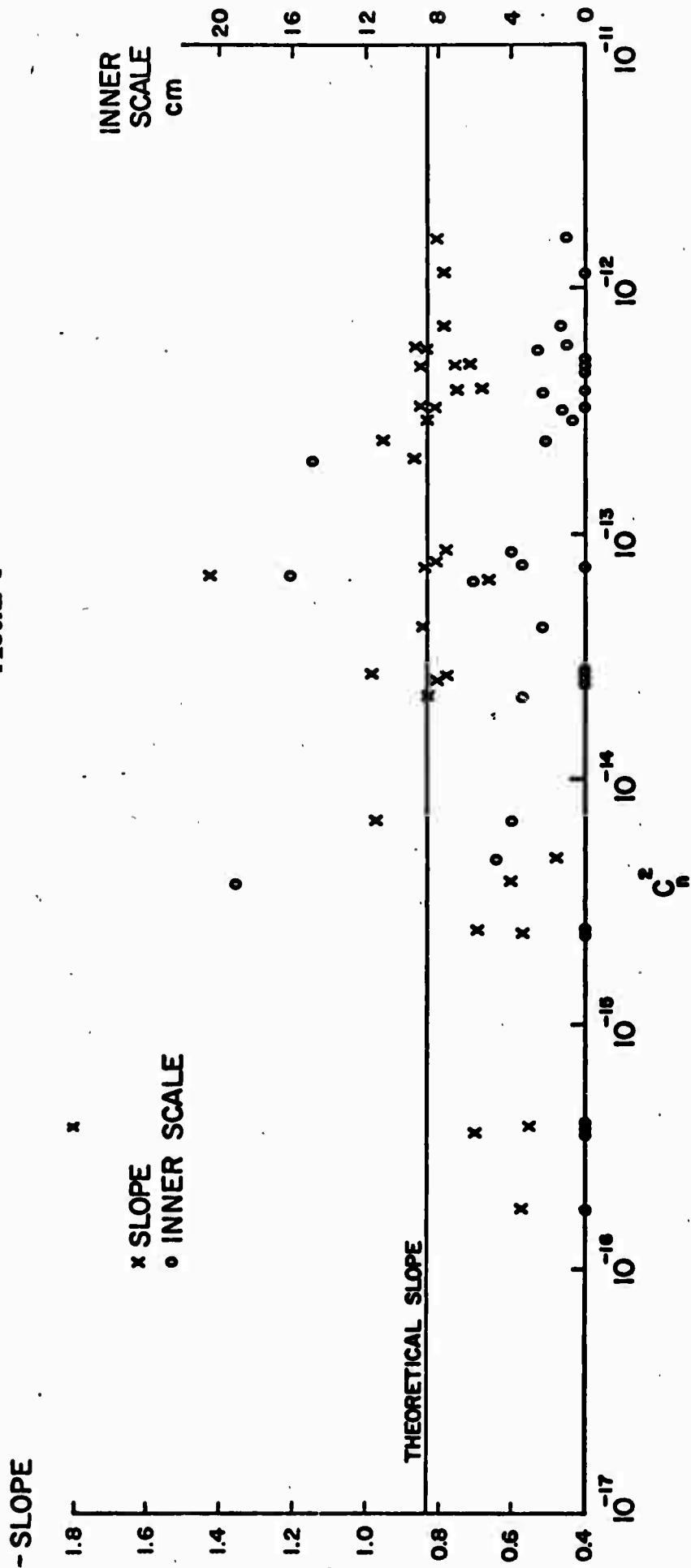
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### Captions to Figures

1. Turbulence spectral (rms) slope and inner scale vs. strength of turbulence.
2. Turbulence spectral slope and inner scale vs. wind speed.
3. Experimental vs. theoretical log amplitude variance. The circles indicate a relatively good Kolmogorov turbulence spectrum.
  - a.  $4880\text{\AA}$
  - b.  $1.15\mu$
  - c.  $10.6\mu$
  - d. Three wavelengths combined
4. Experimental log amplitude variances for pairs of wavelengths.
5. Inner scales as determined from short-path scintillations measured at  $6328\text{\AA}$  vs. those from microthermal spectrum.
6. Transverse log amplitude correlation length vs. strength of turbulence.
7. Transverse log amplitude correlation lengths for pairs of wavelengths.
8. Ratio of peak frequencies of scintillation spectra for two wavelengths, vs. strength of turbulence (Eq. 1).
9. Ratio of (peak frequency  $\times$  transverse correlation length) for two wavelengths, vs. strength of turbulence.

10. Ratio of log amplitude variances at  $4880\text{\AA}$  for large and small receivers, vs. strength of turbulence.
11. Ratio of log amplitude variances at  $4880\text{\AA}$  for large and small receivers, vs. transverse log amplitude correlation length.
12. Ratio of log amplitude variances at  $4880\text{\AA}$  for large and small receivers, vs. normalized covariance at  $r=4$  cm.
13. Log amplitude variances at three wavelengths for 10 sec averaging times, vs. time.
14. Log amplitude variances and turbulence strengths for 10 sec averaging times, vs. time.
15. Log amplitude variances and turbulence strengths for 10 sec averaging times, vs. time.

FIGURE 1





- SLOPE

1.8

1.6

1.4

1.2

1.0

0.8

0.6

0.4

0

FIGURE 2

x SLOPE  
o INNER SCALE

INNER  
SCALE  
cm

20

15

10

5

0

20

WIND SPEED MPH

16

12

8

4

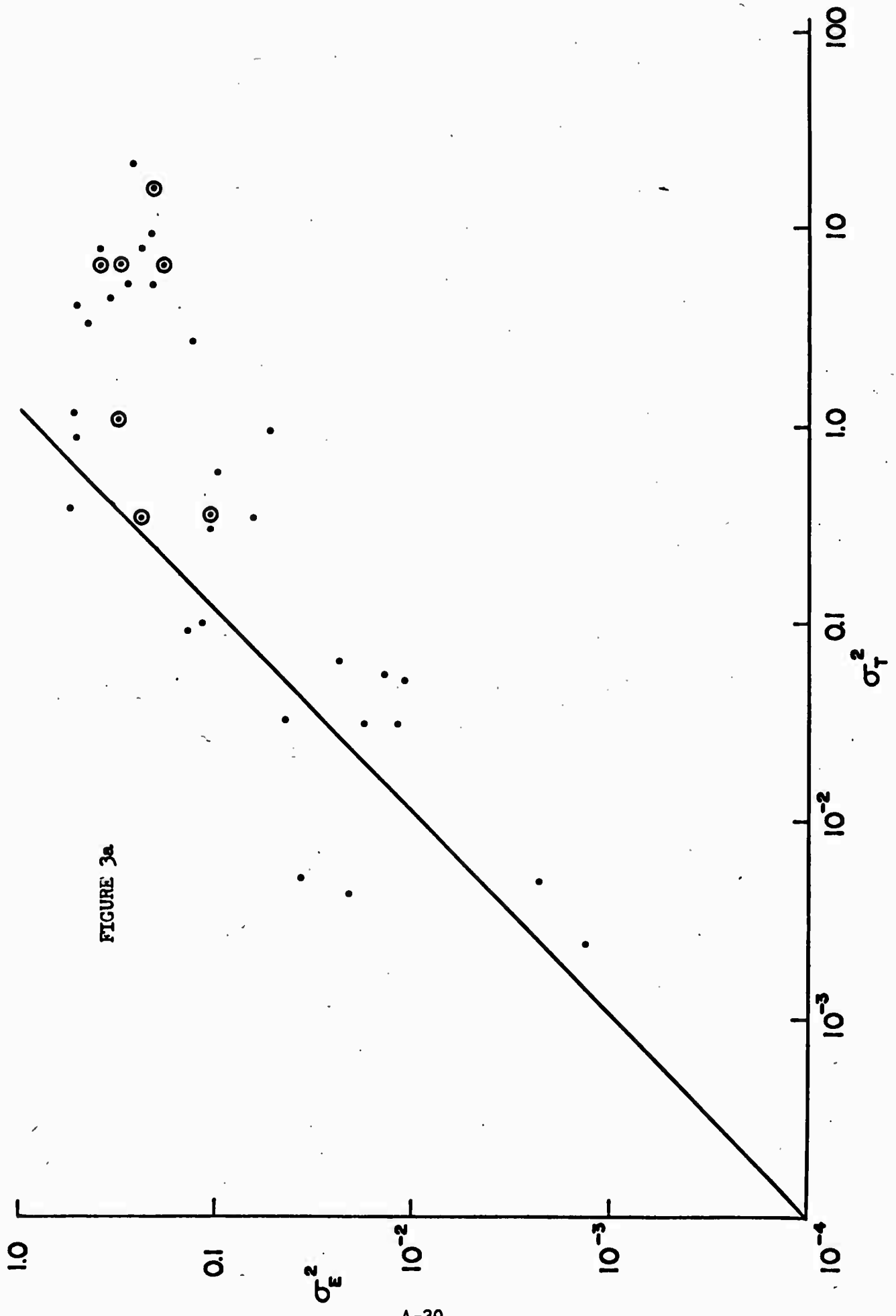
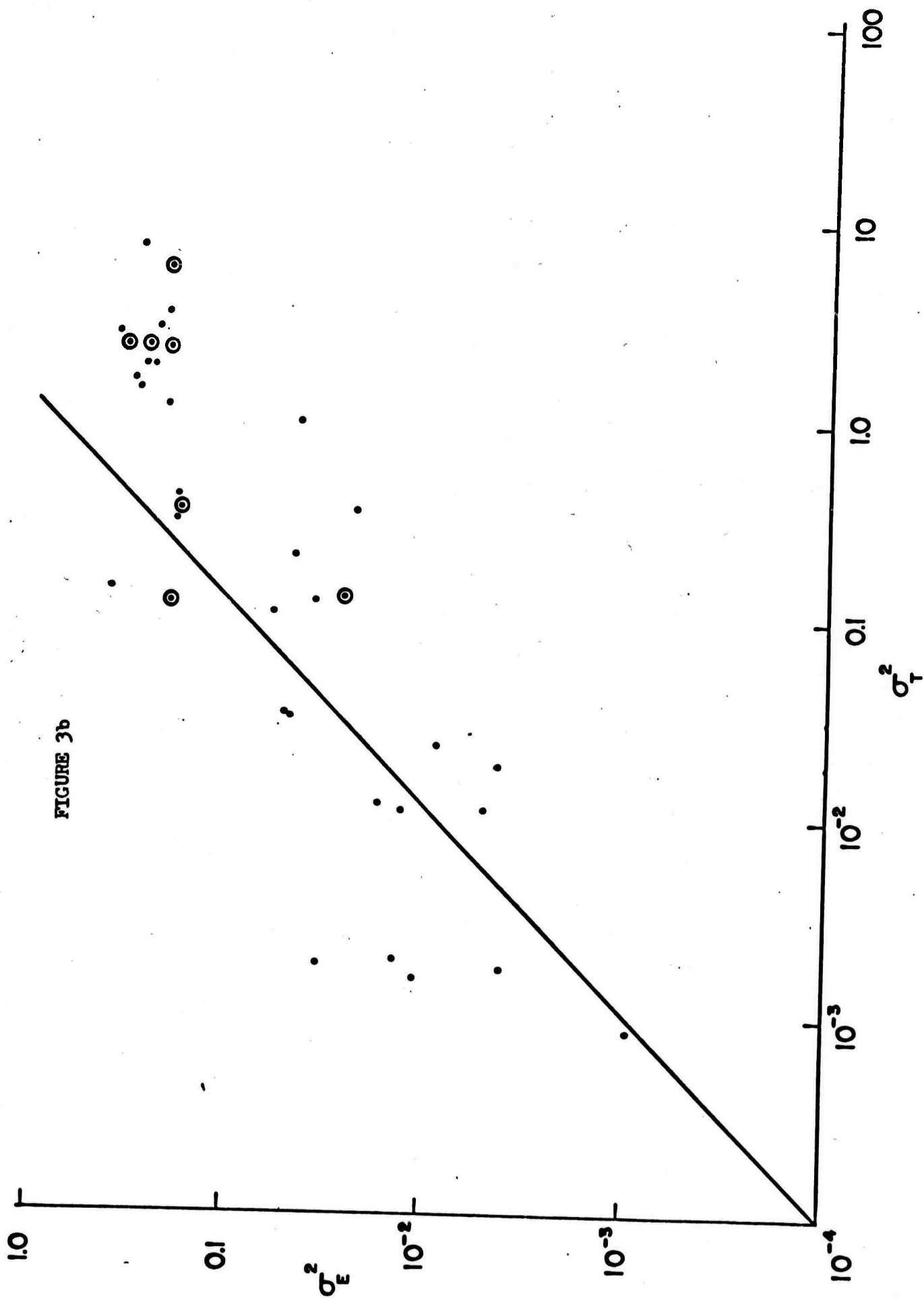
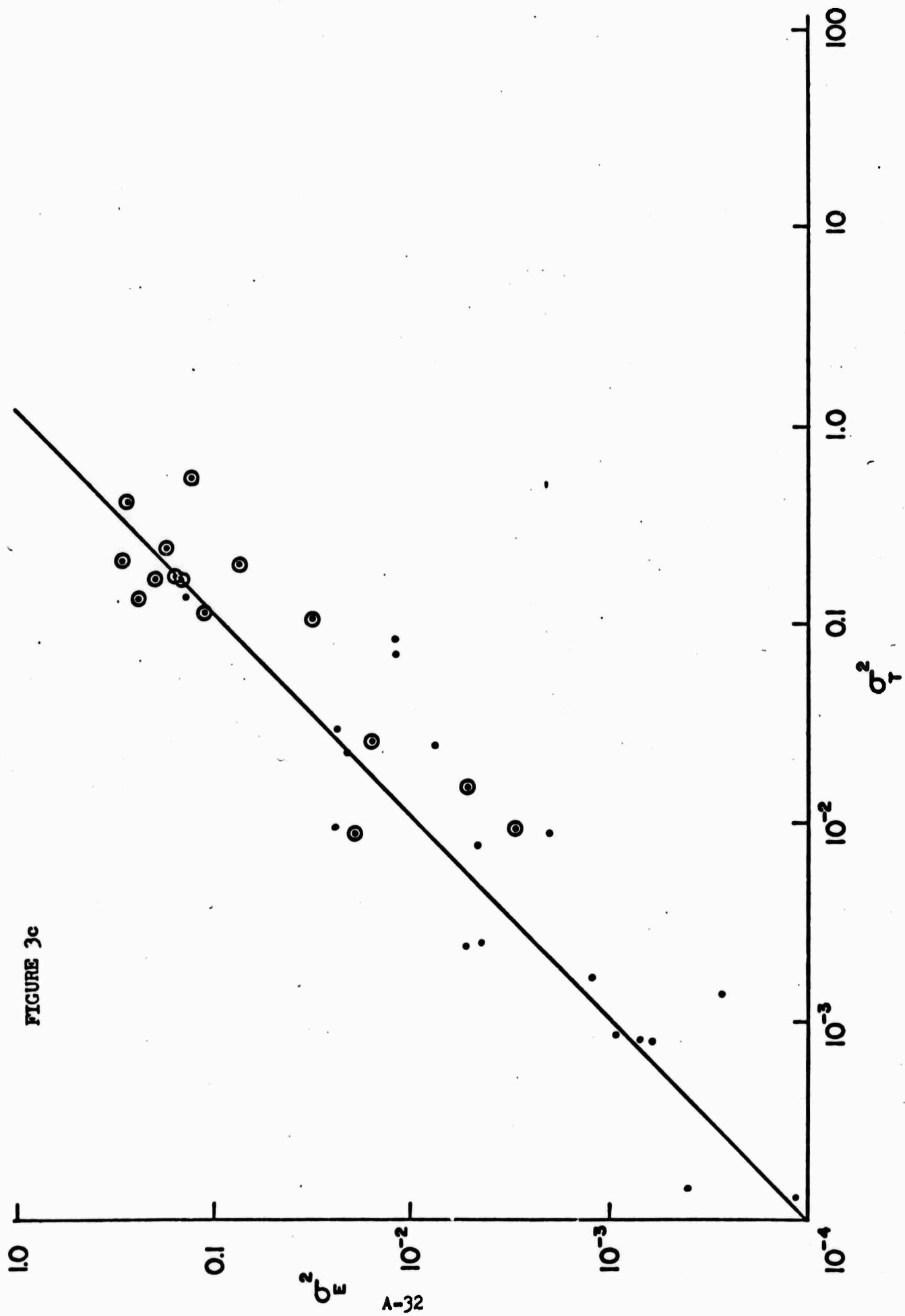


FIGURE 3a





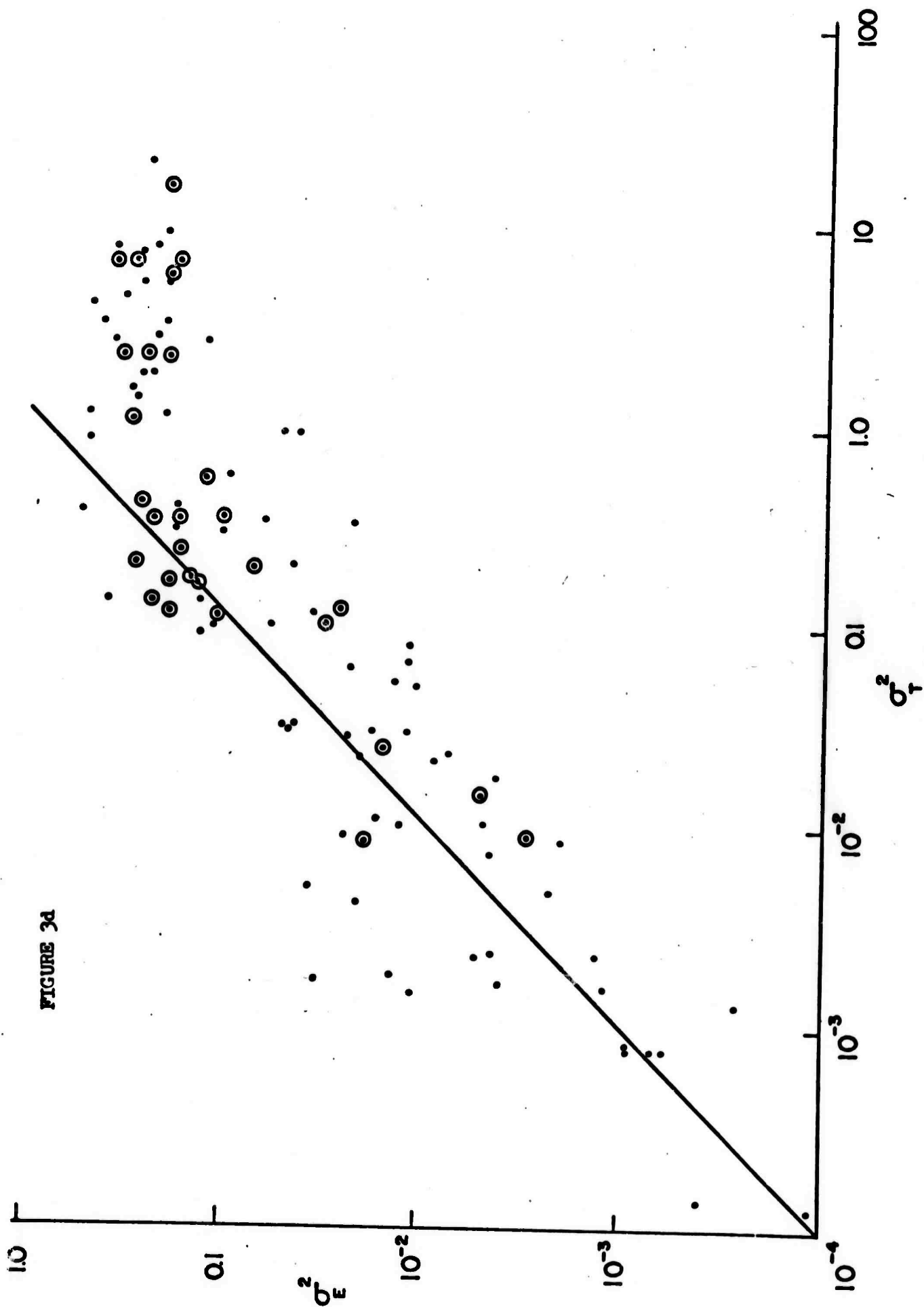
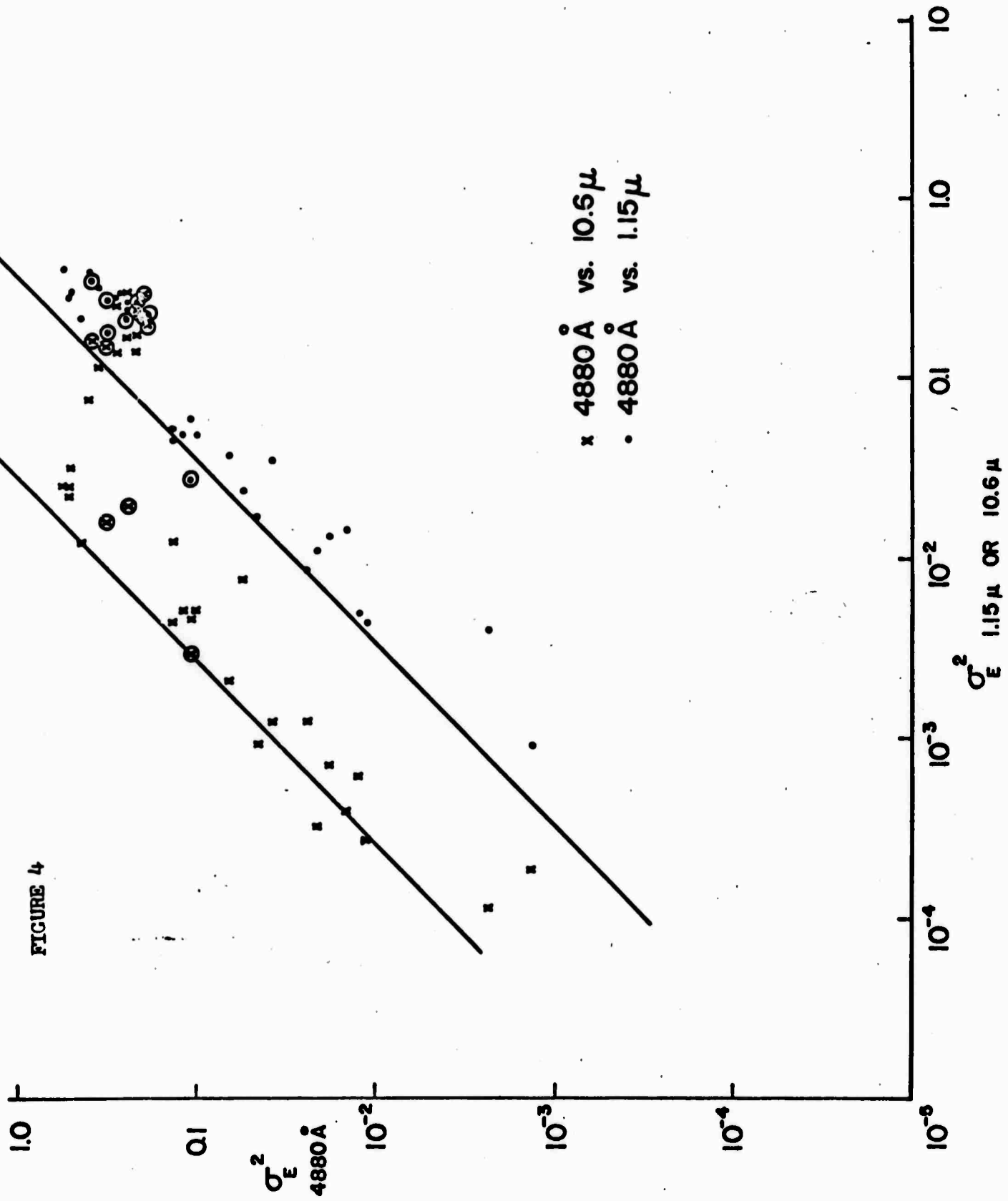


FIGURE 3d



INNER SCALE  
6328 Å

FIGURE 5

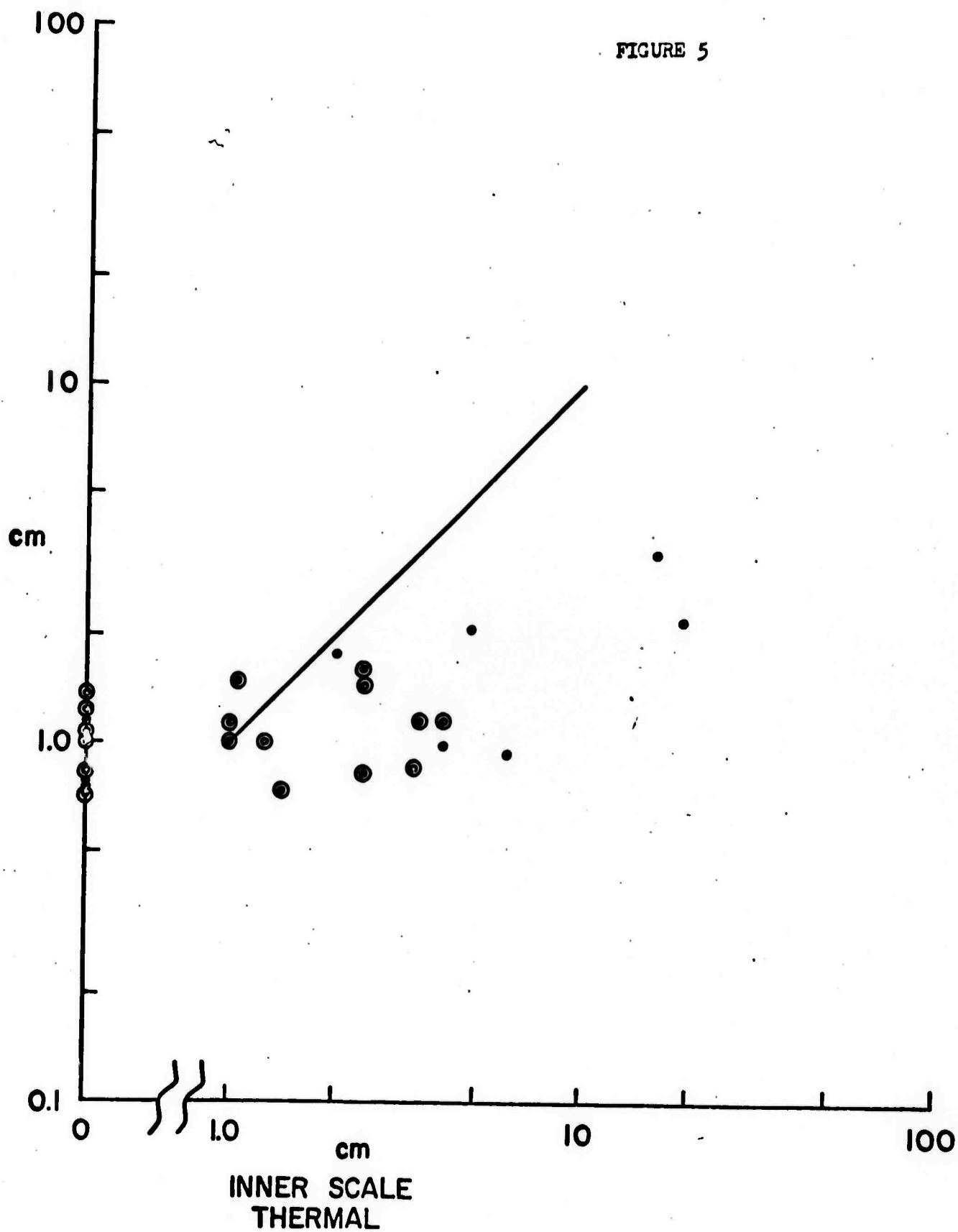
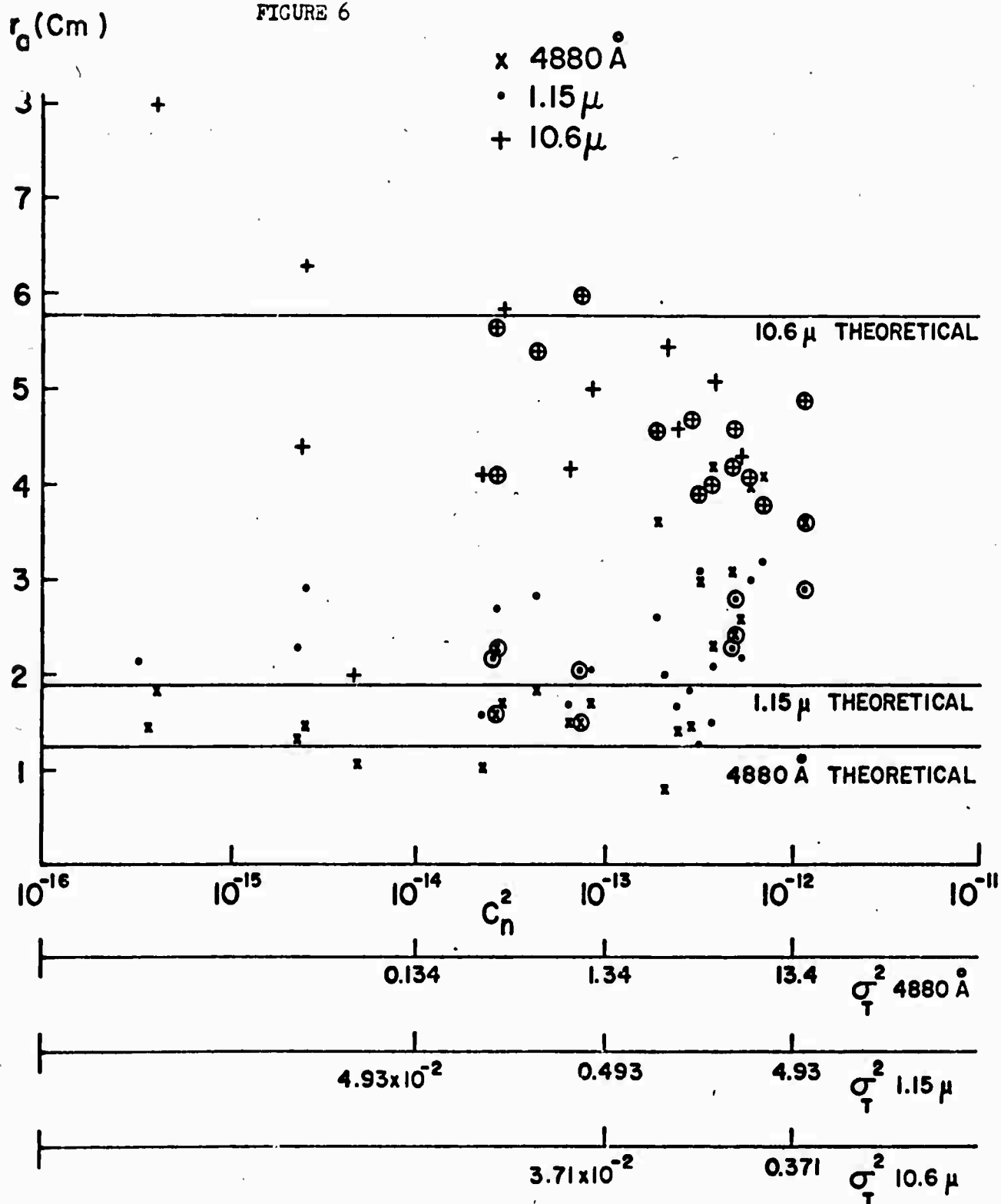


FIGURE 6





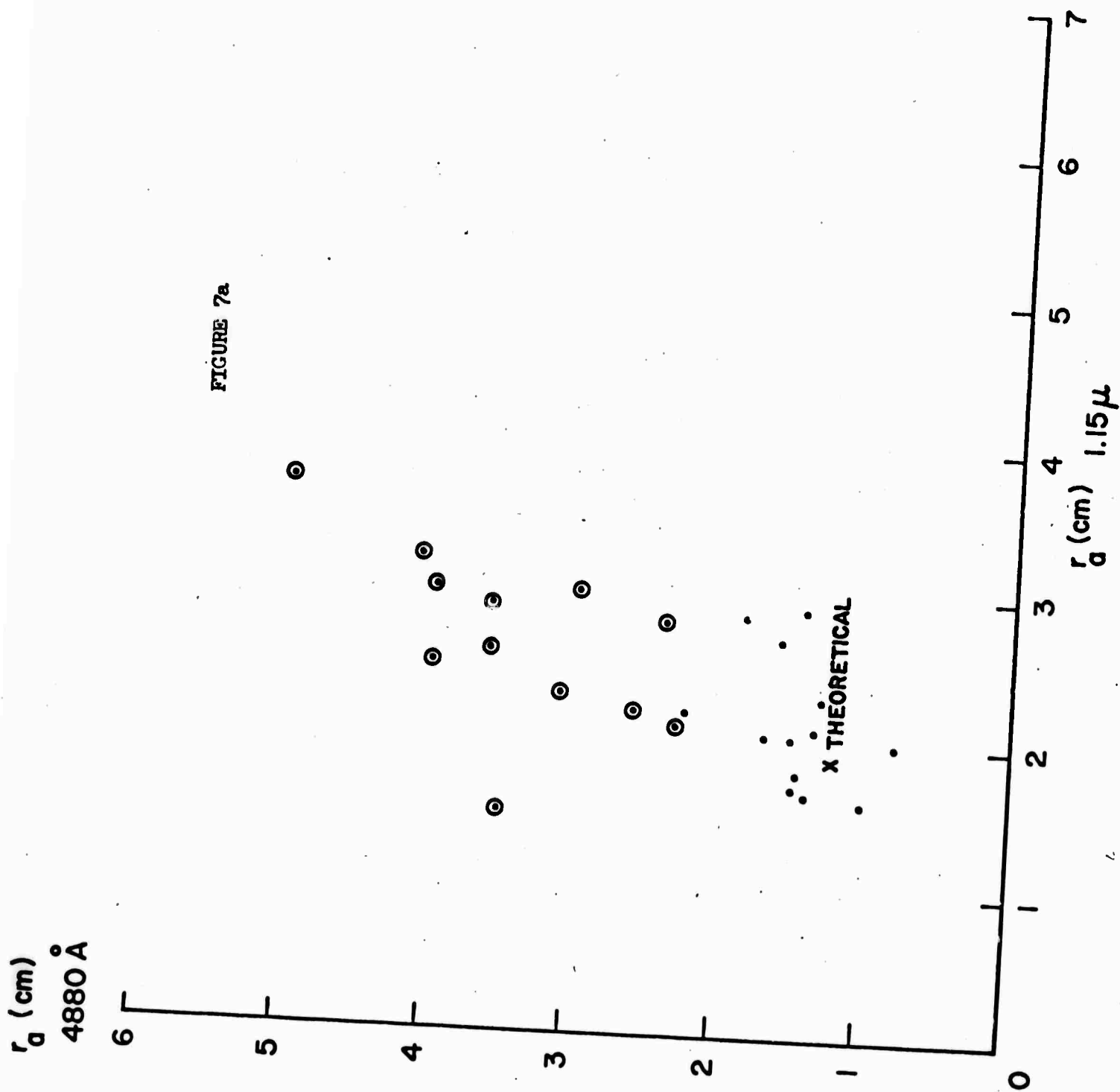
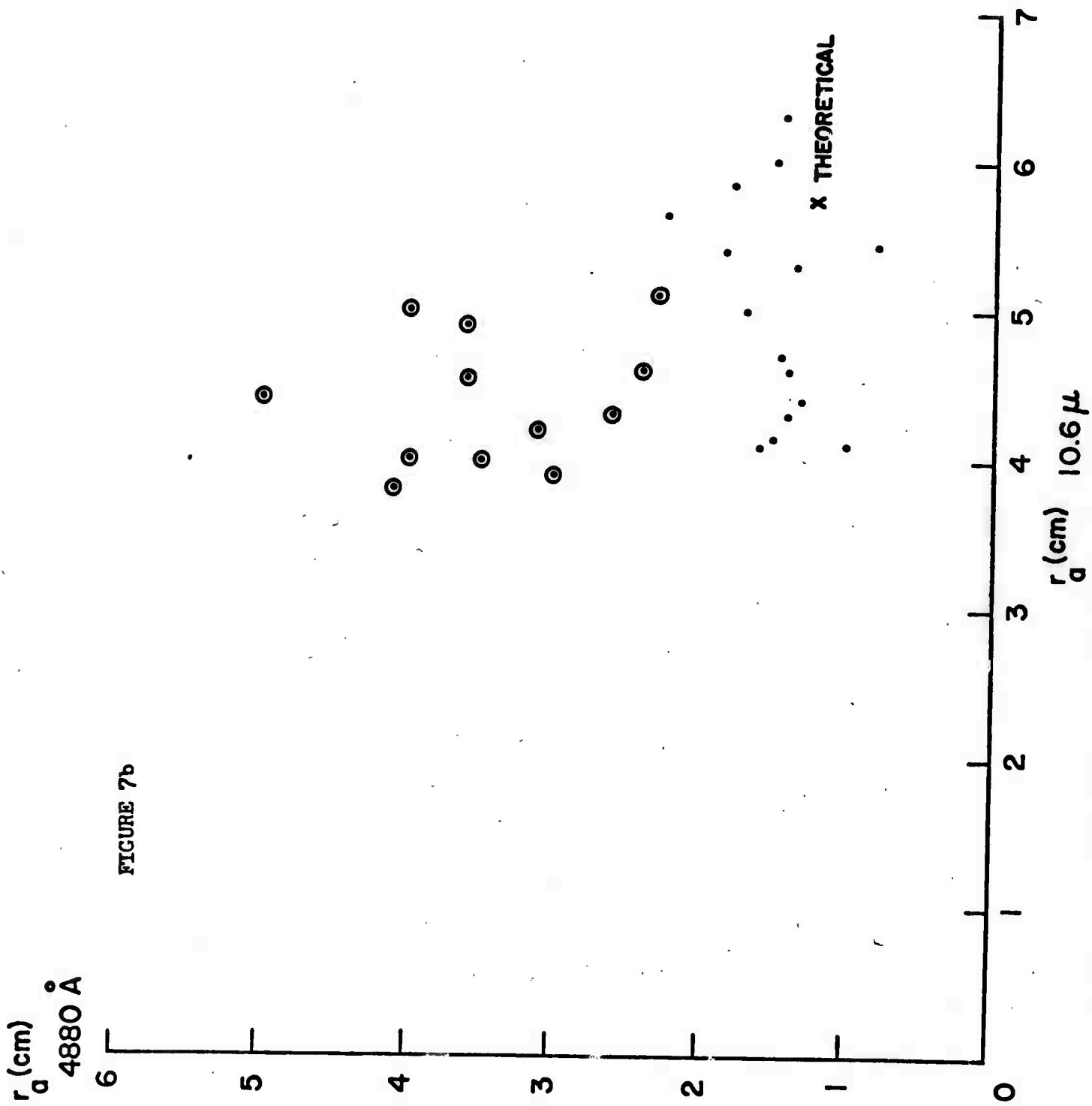
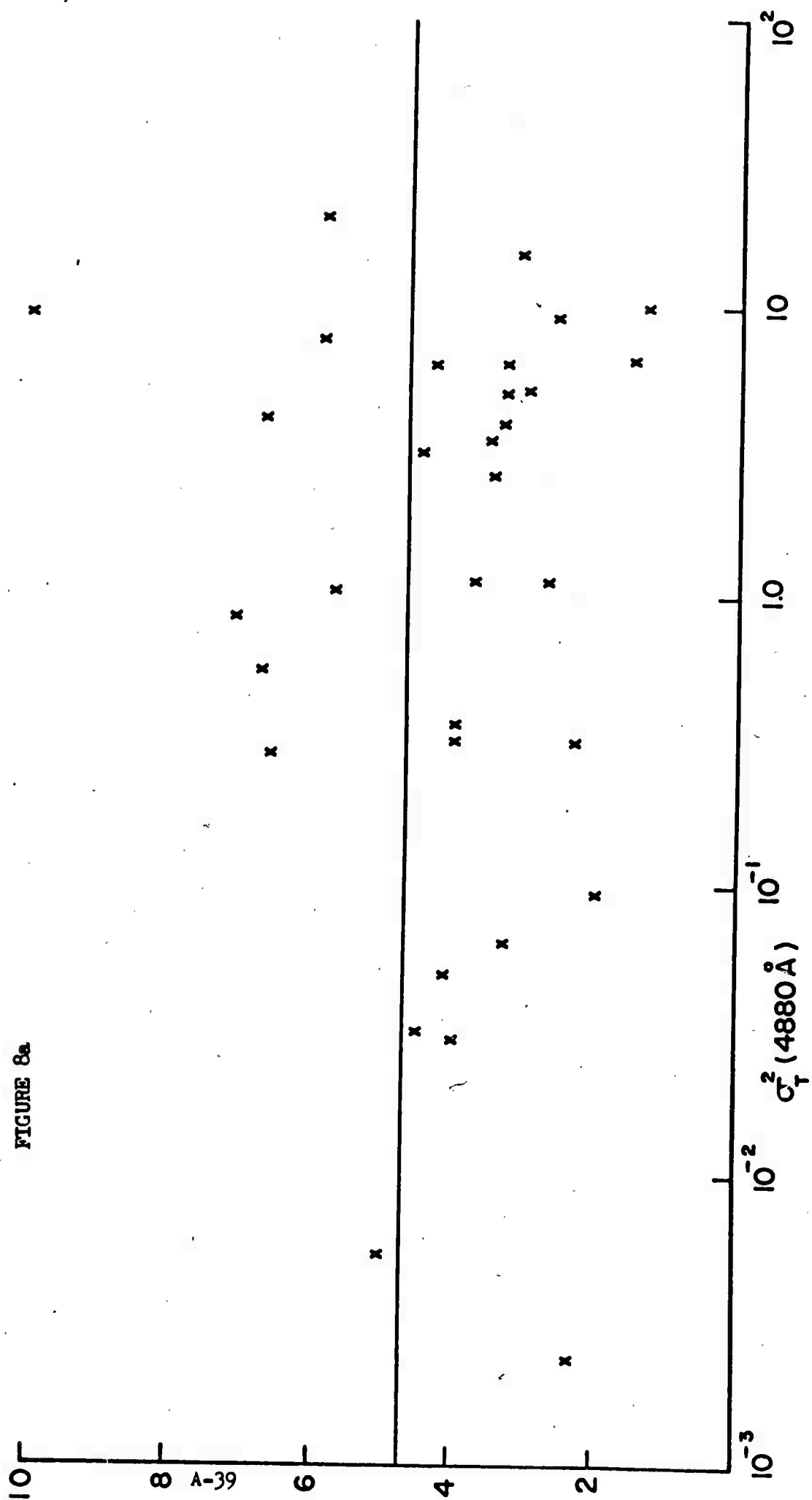


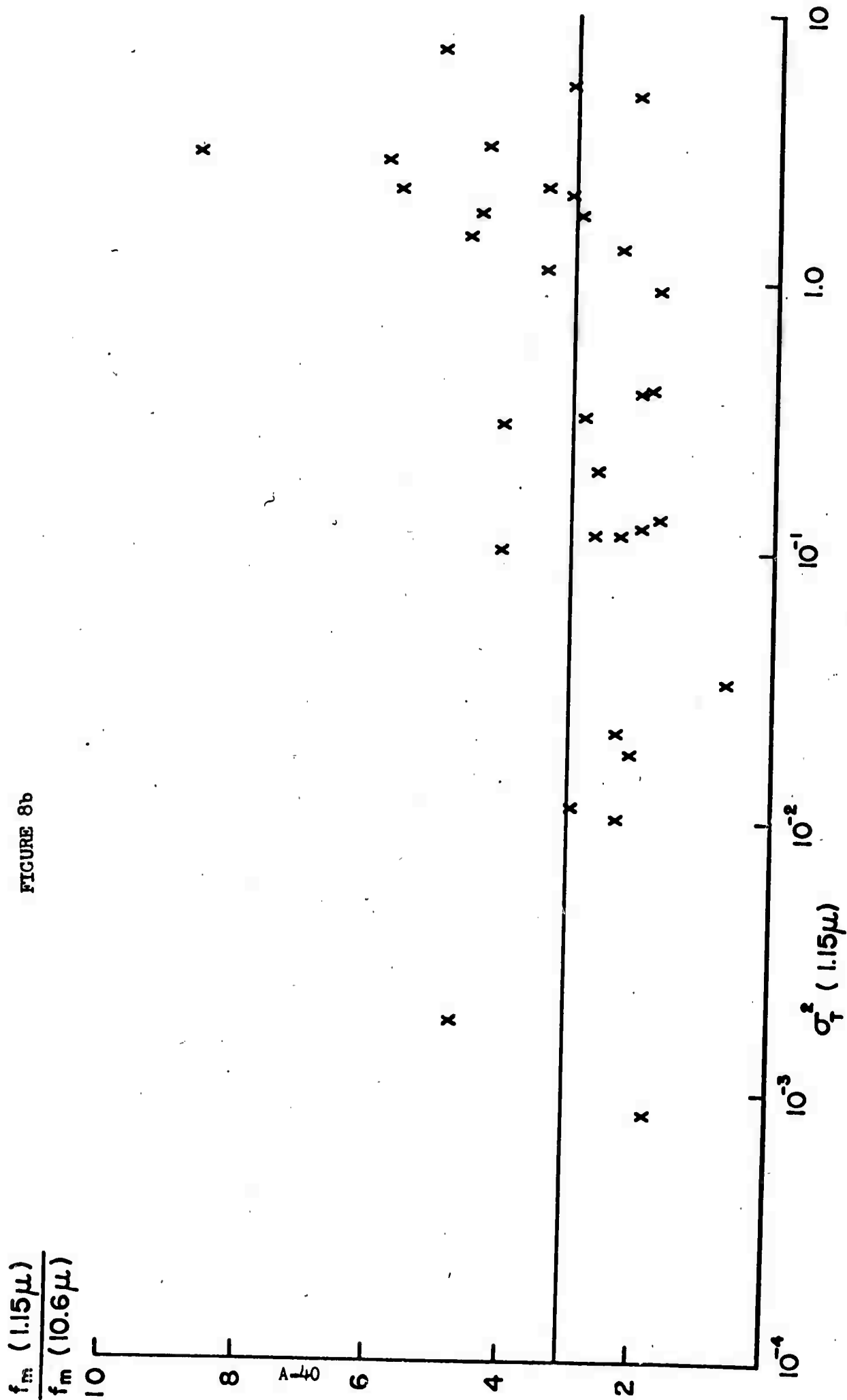
FIGURE 7a

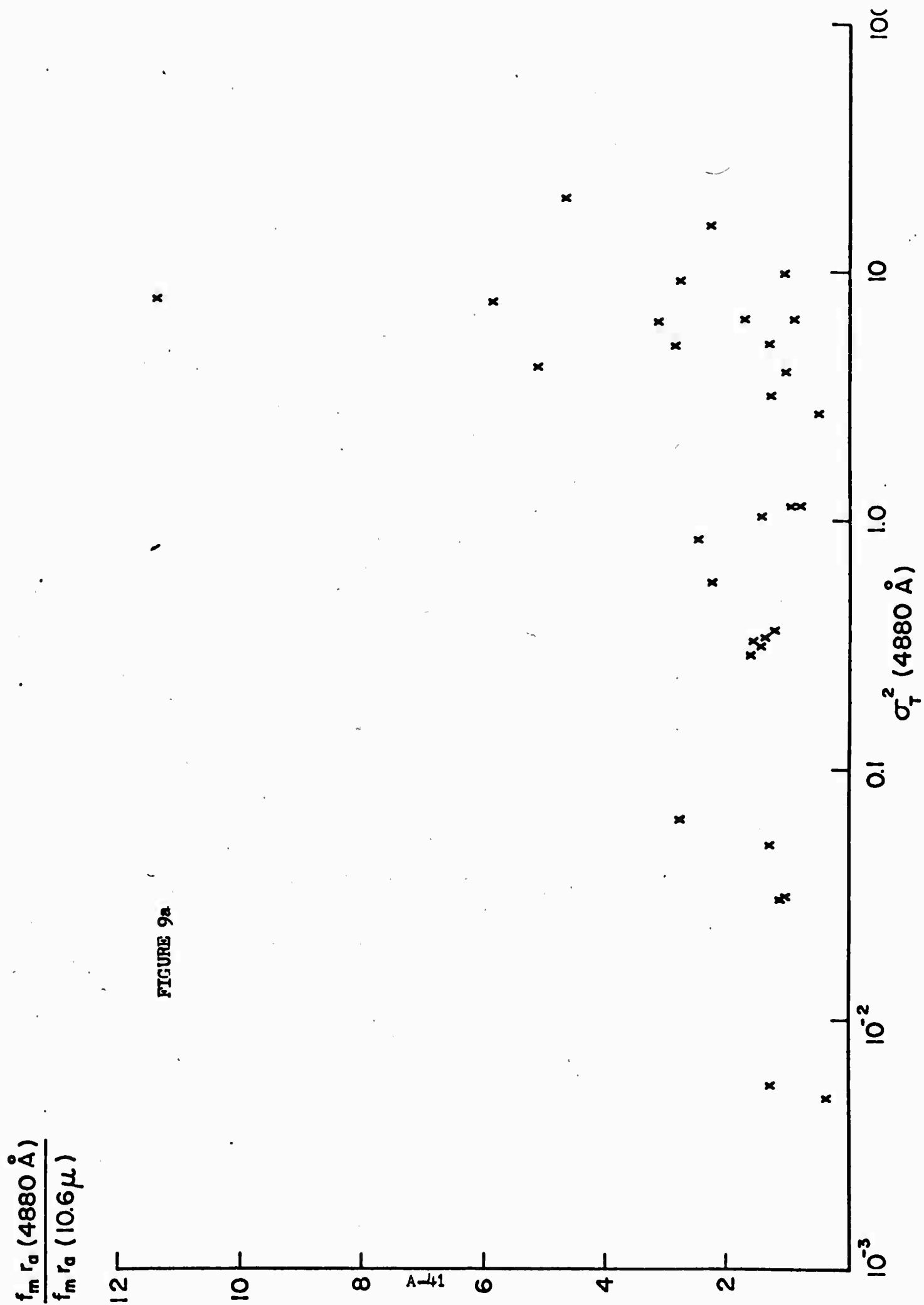


$$\frac{f_m(4880 \text{ \AA})}{f_m(10.6 \mu)}$$

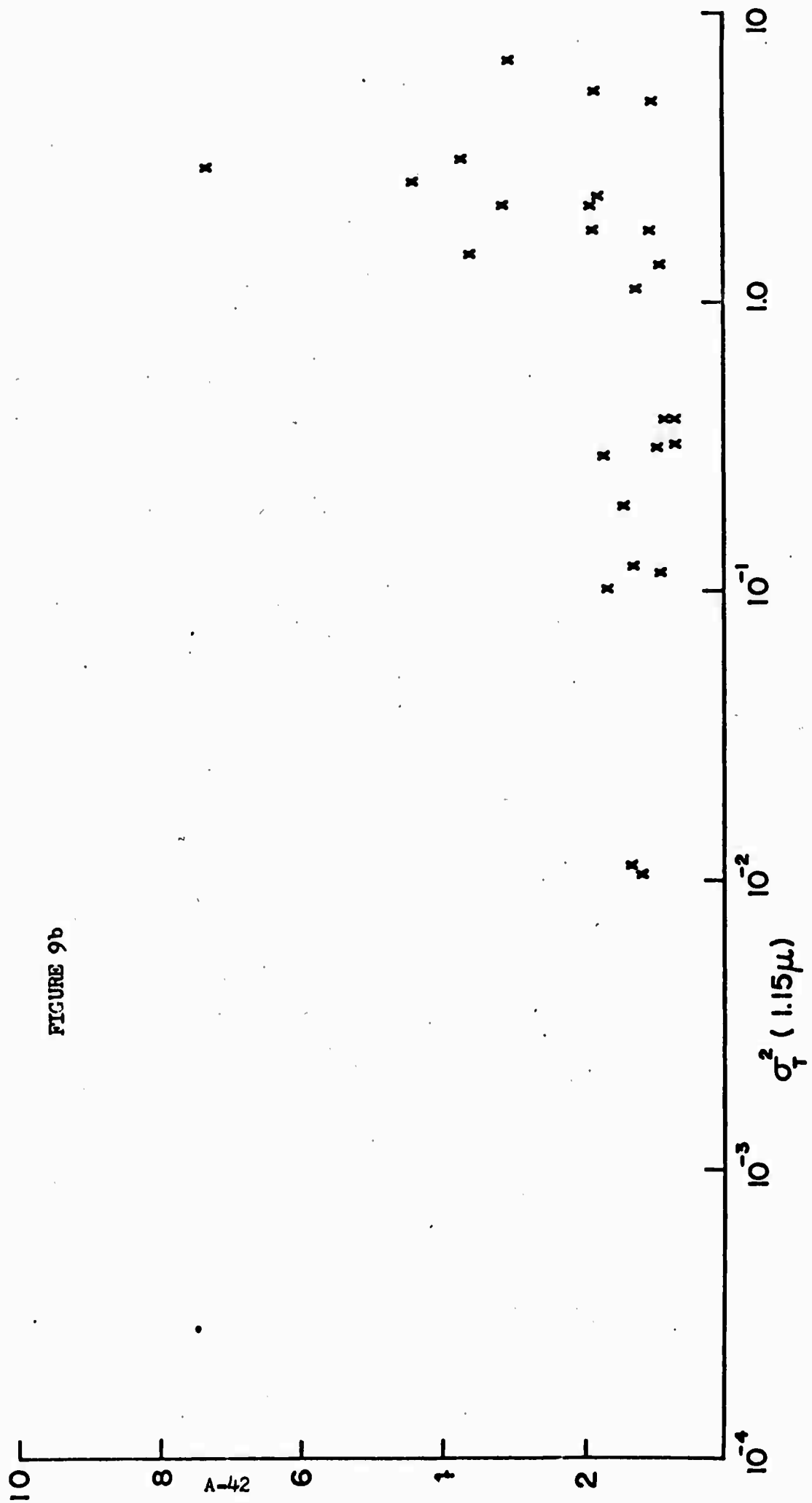
FIGURE 8a

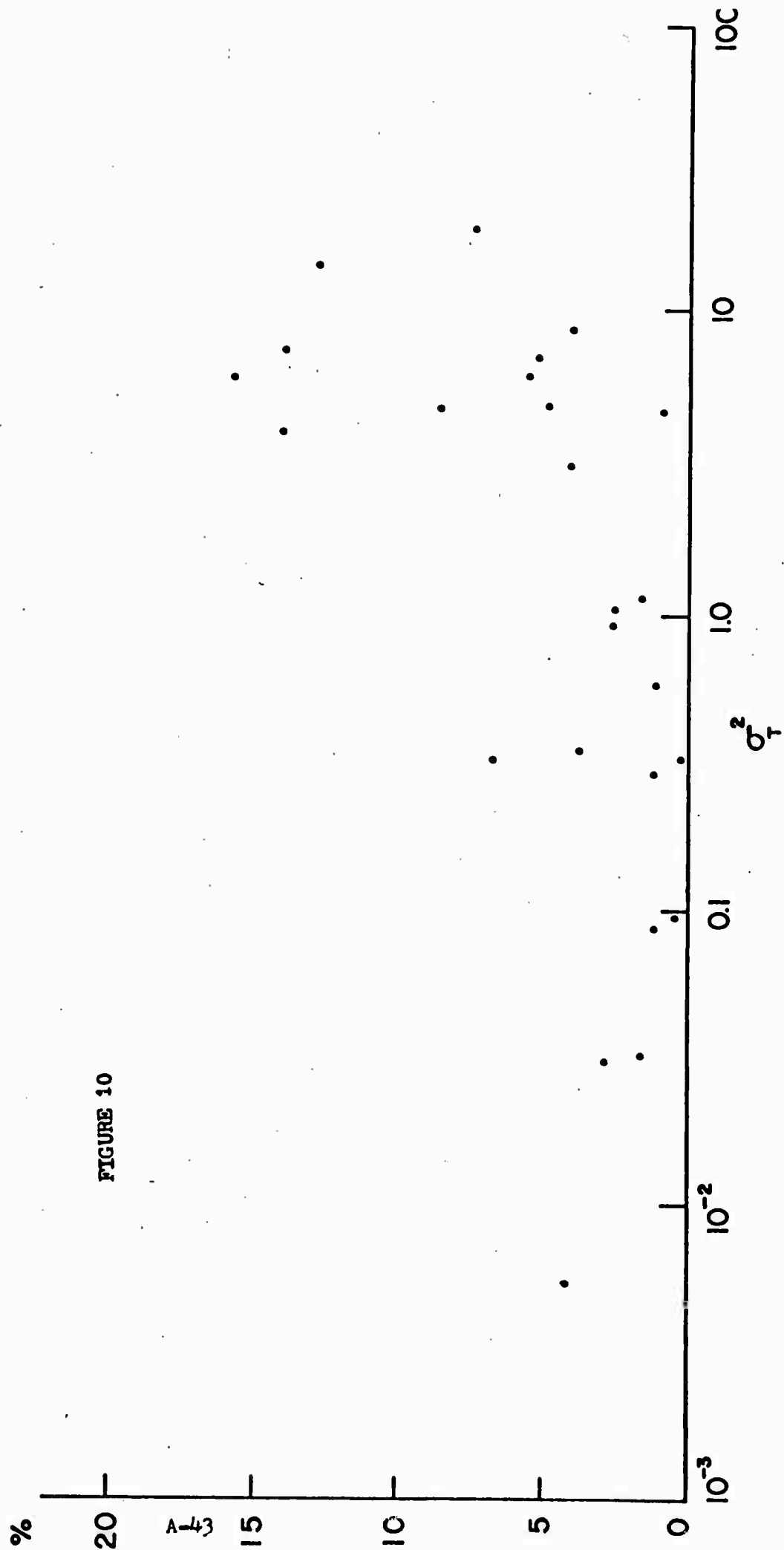


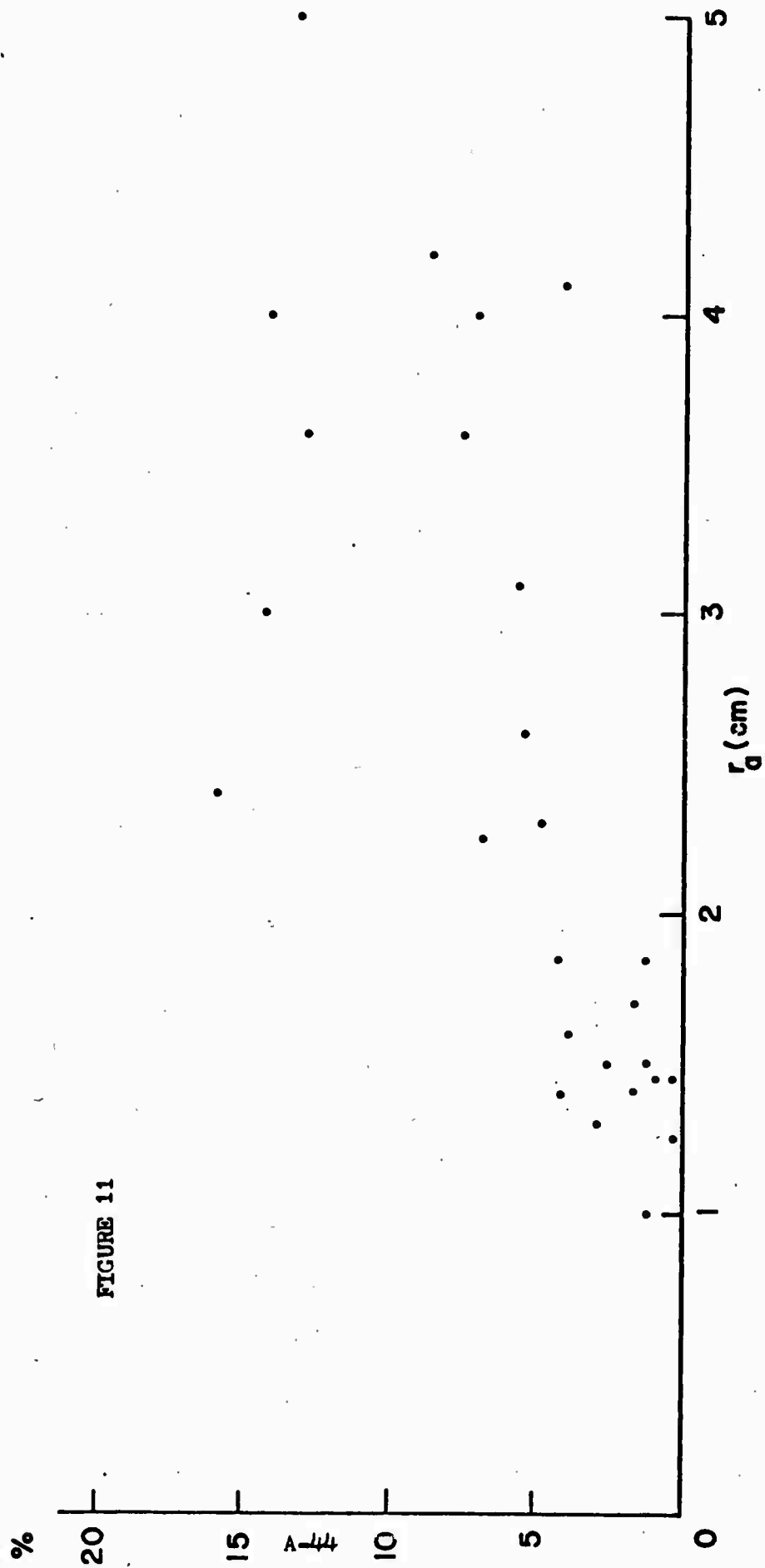




$$\frac{f_m r_a(1.15\mu)}{f_m r_a(10.6\mu)}$$









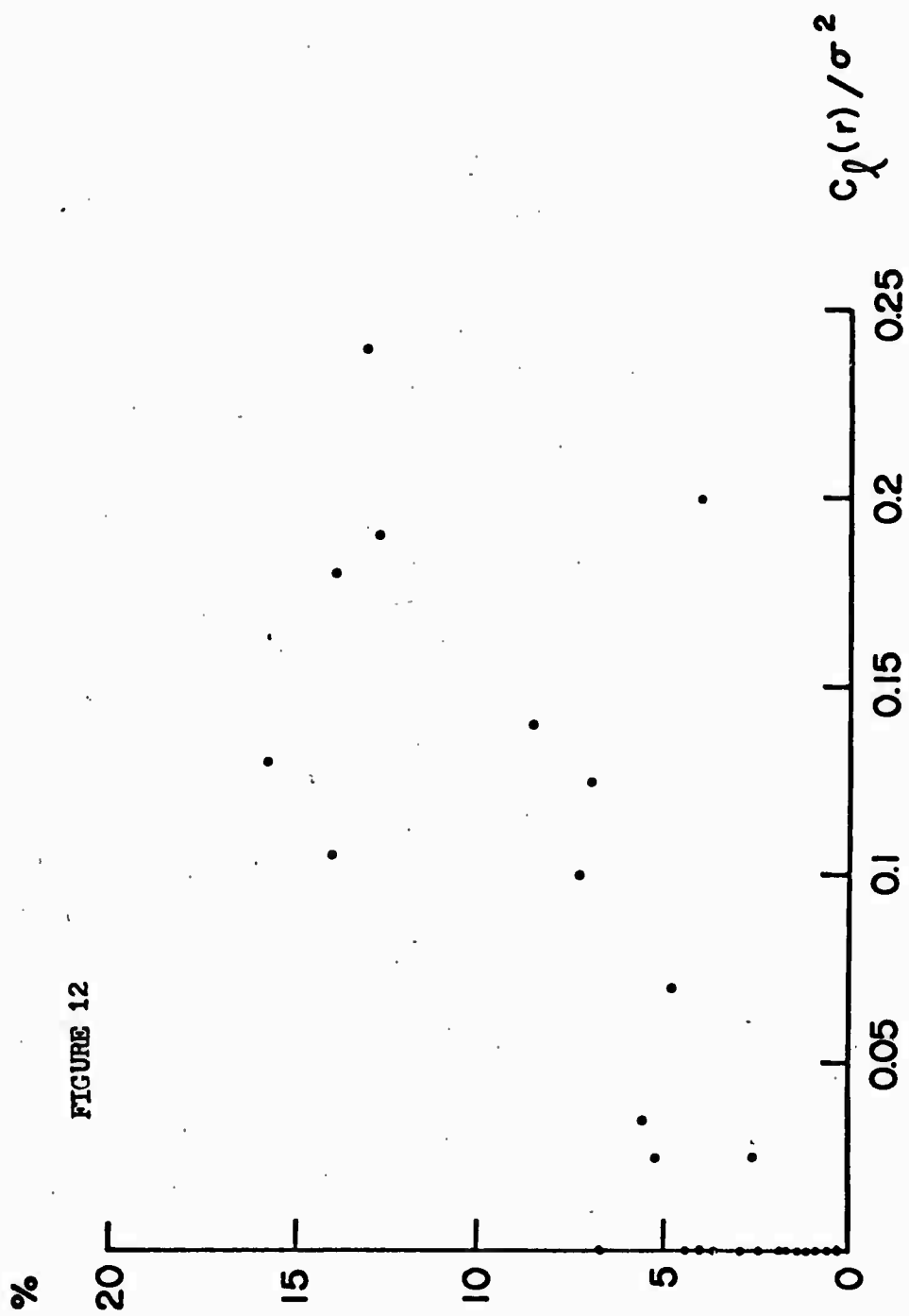


FIGURE 12

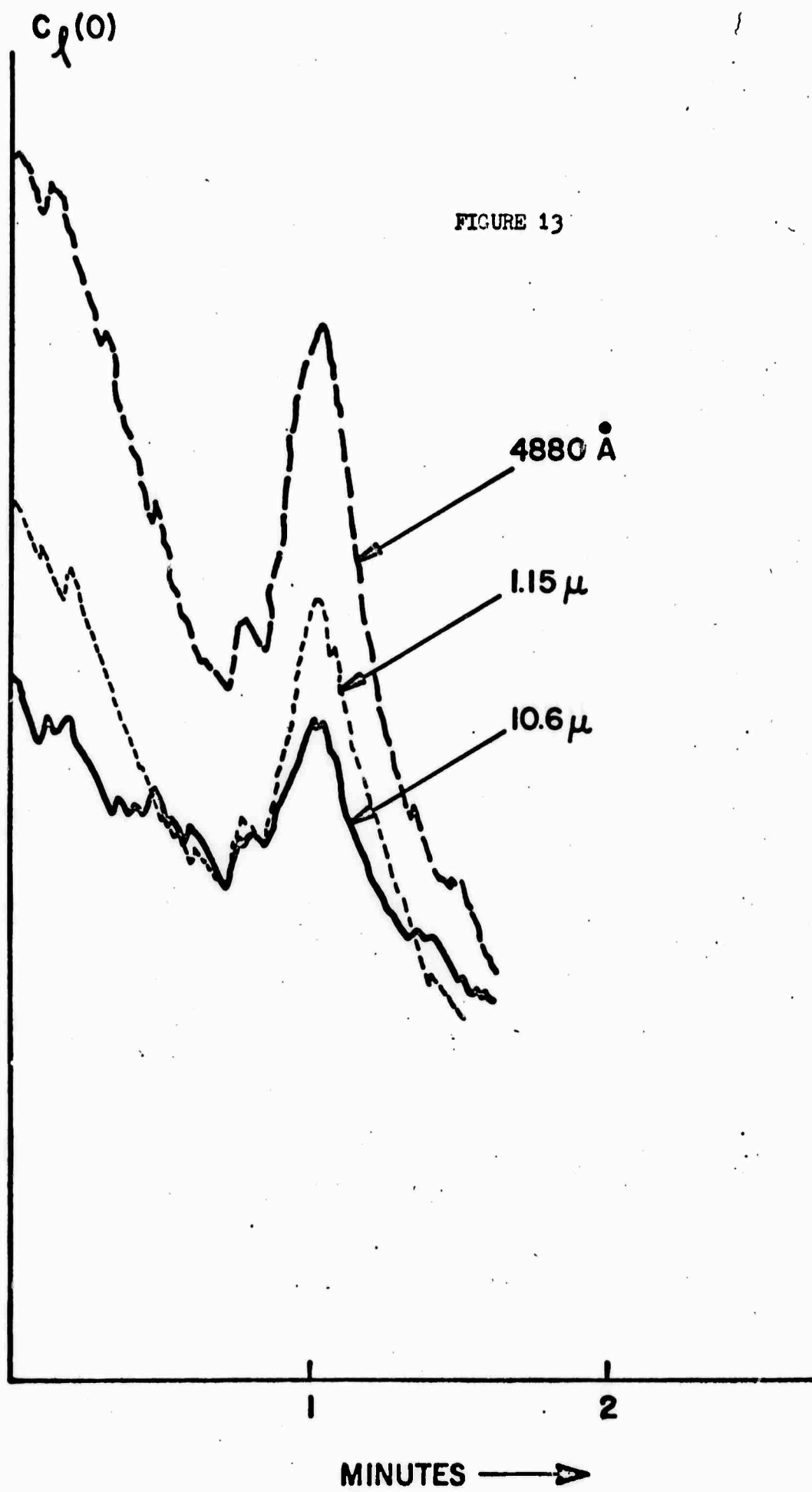


FIGURE 14

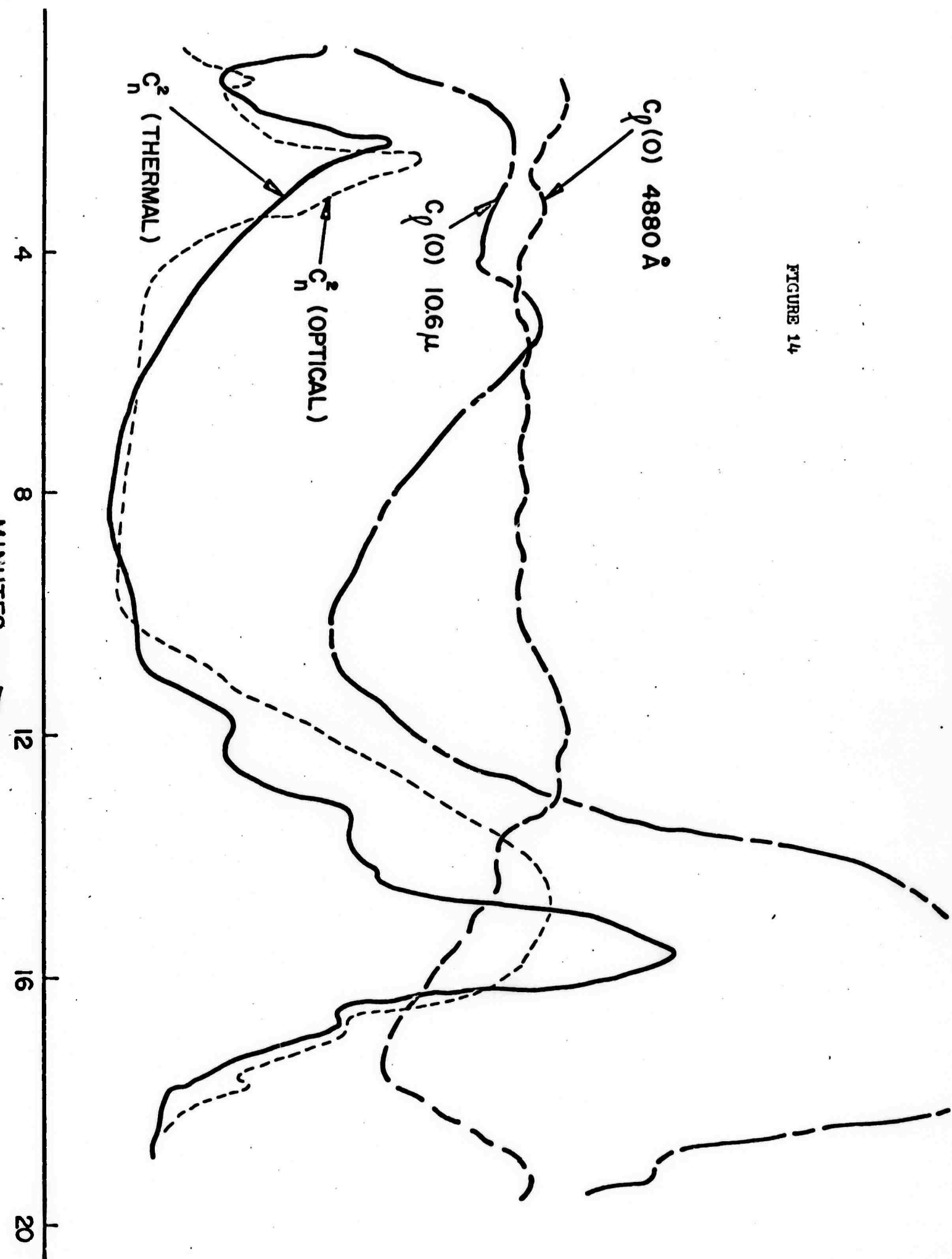
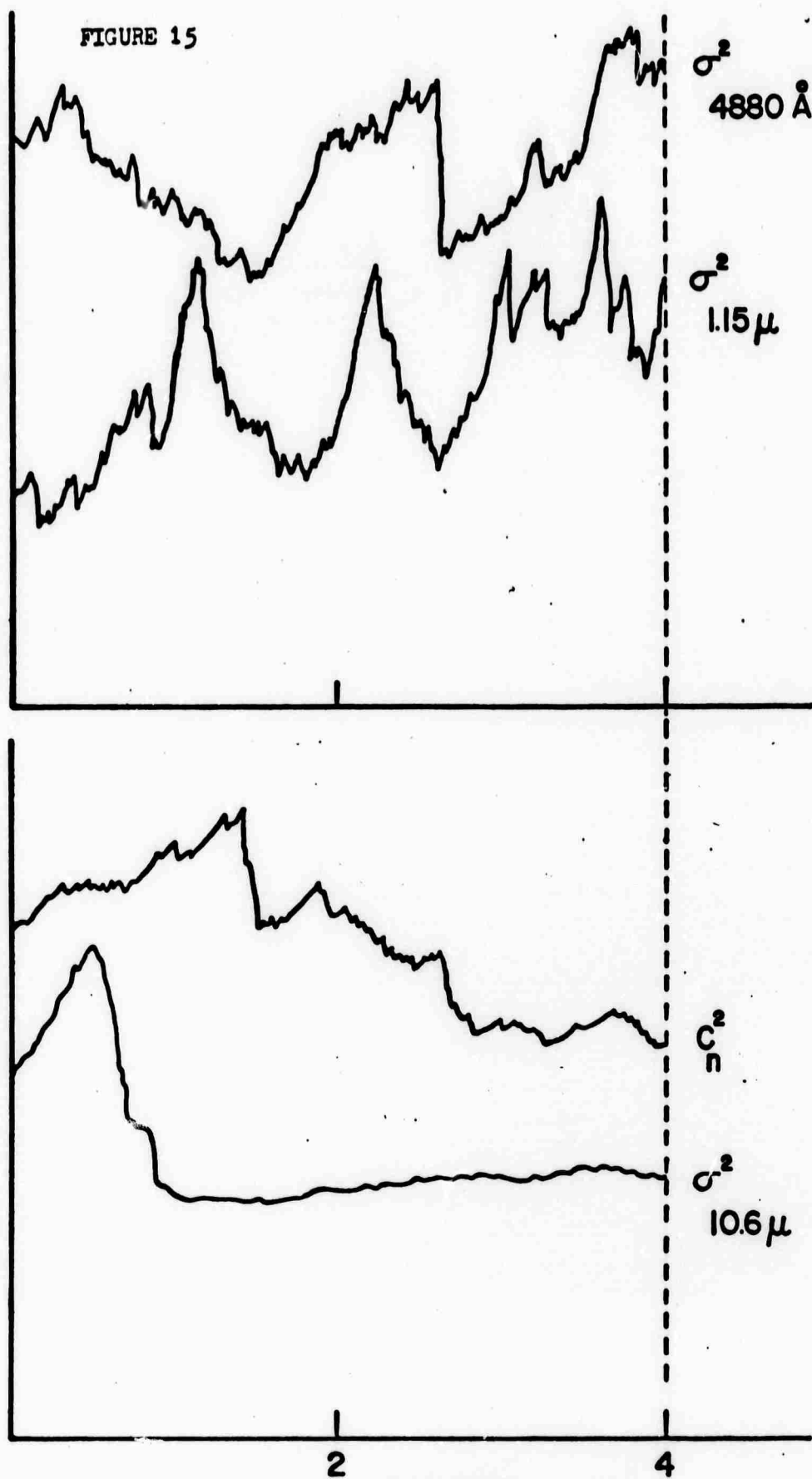


FIGURE 15



MINUTES →

A-48

Table I--Experimental Parameters

Path length: 1.4 km  
Path height: 2 meters  
Wavelengths:  $4880\text{\AA}$ ,  $1.15\mu$ ,  $10.6\mu$  (simultaneous and coincident beams)  
Receiver aperture: 3 mm  
Receiver dynamic range: 80 dB  
Transmitter Fresnel number:  $< 10^{-2}$  (virtual point sources)  
Auxiliary short-range system:  $6328\text{\AA}$  at 165 m  
Aperture-averaging receiver: 32 cm at  $4880\text{\AA}$

Table II--Experimental Measurements

Microthermal:  
    Turbulence spectrum (one-point temperature fluctuations)  
    Inner scale  
    Strength of turbulence (two-point differential temperature fluctuations)  
Multiwavelength scintillation statistics:  
    Log amplitude probability distribution  
    Log amplitude variance  
    Log amplitude covariance function  
    Spectra of scintillations  
    Receiver aperture-averaging  
    Short-path log amplitude variance  
General meteorological parameters

## APPENDIX B

### Intermittency of Turbulence in Atmospheric Scintillation Phenomena \*

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The fundamental intermittency of turbulence, including the conditions of a uniform terrain with constant solar flux, has recently been widely recognized by atmospheric physicists.<sup>1-5</sup> The effect, which is dependent on Reynolds number,<sup>5,6</sup> is related to random dissipation rates at small scales, and necessitates a small correction in the Kolmogorov spectrum.<sup>1,5</sup> More importantly, it results in an intermittent and spiking behavior for parameters such as the temperature fluctuations at a point, and in significant variations in the short-term level of scintillations for a propagating laser beam.

It is often observed that regions of warmer air, containing highly turbulent eddies, are interspersed with cooler regions of relatively low turbulence.<sup>7</sup> One description is in terms of intact "plumes" of warmer air.<sup>8-12</sup> Although most observations have been near the ground, the effects are also seen at several kilometers of altitude.<sup>7,13</sup>

The intermittency has lead to bimodal approximations for the temperature probability distribution.<sup>7</sup> More precisely, the tempera-

ture and velocity derivative fluctuations are log normal in nature,<sup>1-3,5,6</sup> where a bilateral form should be used when the variable has a zero mean.<sup>6</sup> The nonuniformity of the turbulence may also extend to its spatial spectrum.<sup>14</sup>

Amplitude scintillations are primarily affected by small scale turbulence components, which are in turn affected by various scales of intermittency. If we designate the outer scale of turbulence by  $L_0$ , the scales of intermittency or inhomogeneity may be distinguished as follows:

1. Large scale ( $\gg L_0$ ) regions in which medium and small scale turbulence components are increased
2. Medium scale regions ( $\approx L_0$ ) in which small scale turbulence components are increased
3. Fine inhomogeneities, such as the sharp boundaries which are observed between larger regions

The intermittencies render Fourier analysis awkward,<sup>15</sup> and suggest the need for a theory for short-term atmospheric and scintillation phenomena and measurement.<sup>16</sup>

Examples of probability distributions of microthermal fluctuations ( $\Delta T$ ) are shown in Figure 1. A small, single probe was used with low-noise electronics,<sup>17</sup> and the points on each curve were taken sequentially over a period of about two minutes. The straight line (gaussian) distribution was observed during a uniform period of turbulence, while the double gaussian with a break-point indicates an

abrupt (large-scale) change. The third curve, with pronounced tails, is associated with a more characteristic spiking behavior. Such a distribution is shown for  $\log (\Delta T)^2$  in Figure 2, indicating near log-normality.

Optical/infrared scintillations are determined by a weighted average of the high and low turbulence regions along the path. Thus, large-scale inhomogeneities are not well averaged and give rise to the familiarly large spread in scintillation data; it has been estimated that this effect greatly overrides the spread due to variations in the turbulence spectrum.<sup>18</sup> It is thus suggested that quantities such as the short-term variance of log amplitude  $[\sigma^2(t)]$  and refractive index structure constant  $[C_n^2(t)]$  be treated as related random variables. Theory then needs to be developed to describe this relationship, and to predict data spread and confidence limits. It may be noted that inhomogeneities, whether systematic<sup>19</sup> or random, will also affect the covariance of scintillations, and hence the effectiveness of receiver aperture smoothing. Also, fluctuations in  $\sigma^2(t)$  imply deep signal fading which is obscured in long-term averages.

A related question is that of stationarity vs. longer averaging times.<sup>20</sup> For practical purposes, stationarity may be said to fail when no choice of averaging time for  $\sigma^2(t)$  or  $C_n^2(t)$  will result in consecutive values which are consistent (small spread), and free of monotonic (including diurnal) trends. This may happen with a low



wind or under changing meteorological conditions. Conversely, uniform conditions, reasonable winds, and/or longer paths result in less data-spread for shorter averaging times.<sup>21,22</sup>

To indicate how a theoretical development might proceed, let us assume that there is no systematic variation in turbulence along the path (long-term homogeneous). If we represent the instantaneous large-scale turbulence profile by the random variable  $C_n^2(z,t)$ , we may write the log amplitude variance for a point source using the expression for a nonuniform, smoothly-varying turbulence level:<sup>23</sup>

$$\sigma^2(t) = 0.14 k^{7/6} \int_0^L C_n^2(z,t) (z/L)^{5/6} (L-z)^{5/6} dz, \quad (1)$$

where  $z$  is the distance from the transmitter, and  $k$  is the optical wavenumber. With a sufficient knowledge of the statistics of  $C_n^2(z,t)$ , the statistics of  $\sigma^2(t)$  may be calculated for a given averaging time. The long-term average will of course yield the usual expression<sup>23</sup>

$$\sigma^2 = 0.12 k^{7/6} L^{11/6} C_n^2. \quad (2)$$

The presence of large-scale regions of turbulence with abrupt boundaries suggests that the assumption of local homogeneity<sup>23</sup> may be poorly justified. In such a case, Eq. (2) may not give the correct long-term average, and a fundamental modification of the propagation theory is required. To illustrate how such a situation may apply, we consider the following extreme case. Suppose that there

exists an optical path which is instantaneously quiescent over all but a region of length  $L'$  near the receiver. We assume that  $l_0 > (\lambda L')^{1/2}$ , where  $l_0$  is the inner scale of turbulence; this is often the case for e.g. 200 meters of path length or less.<sup>14</sup> We must then apply a geometrical optics expression to obtain  $\sigma^2$ ,<sup>23</sup>

$$\sigma^2 \sim C_n'^2 L'^3 l_0^{-7/3} \quad , \quad (3)$$

where  $C_n'$  is the actual structure constant within the highly turbulent region. Now, the true variance as given by Eq. (3) is much smaller than that taken from Eq. (2), where  $C_n^2$  is taken as the time- or path-averaged value ( $C_n'^2 L'/L$ ). This type of situation may be repeated several times over a path.

In recent work<sup>24</sup> deWolf attributes the log normality of scintillations (as opposed to a Rayleigh distribution<sup>25</sup>) and the data spread in experimental measurements, to large-scale intermit-tencies. He also suggests a relationship between the degree of averaging of inhomogeneities (as influenced by wind speed) and the level of saturation of scintillations, a relationship which is borne out by experiments.<sup>23</sup>

In preliminary experiments we have measured the probability distributions and normalized variances of  $C_n^2(t)$  and  $\sigma^2(t)$  taken as short-term averages. The scintillations were measured simultaneously at  $4880\text{\AA}$  and  $10.6\mu$  over a common 1.4 km path. We have also determined the normalized variance of spectral components of  $\Delta T$ , for

a 1 Hz bandwidth centered at 2 Hz and 100 Hz respectively. Representative results are given in Table I, where  $\sigma_t^2$  (4880Å) is the theoretical value of  $\sigma^2$  as obtained from the measured value of  $C_n^2$  and Eq. (2), and indicates the strength of turbulence.

Although general conclusions may not yet be drawn, several possible trends are evident from these and other measurements. As the wind speed decreases, the intermittency of  $C_n^2$  increases drastically. Under highly turbulent conditions, the same is true of scintillations, and the shorter wavelength is better-averaged (less intermittent) than the longer, as would be expected. However, under conditions of weak (and hence possibly poorly-developed) turbulence, scintillations are relatively intermittent in spite of the high wind, and the expected wavelength effects are not observed.

Using one-second averaging for  $C_n^2(t)$  and  $\sigma^2(t)$ , probability distributions of  $C_n^2(t)$  are highly skewed as expected, while those of  $\sigma^2(t)$  are sometimes skewed but tend to be gaussian in about half the cases examined. Temporal records of these quantities for Run No. 2 are shown in Figure 3.

In all cases, the spectrum of temperature fluctuations is more variable at lower frequencies than at high. Typical narrow-band fluctuations are shown in Figure 4, centered at 2 Hz and 100 Hz respectively.

In conclusion, it is apparent that in order to quantitatively

describe the nature of turbulence intermittence and its effects on scintillation statistics, levels, and data spread, further theoretical efforts are needed, coupled with experiments in which measurements of the type illustrated above are made under a wide variety of conditions. Other parameters such as the autocorrelation of  $C_n^2(t)$  are also pertinent. The experimental facility should include microthermal, wind, solar flux, and lapse-rate instrumentation at a number of points. In particular, it is desirable to take data during periods of highly-homogeneous conditions over the path,<sup>18</sup> and to relate the degree of homogeneity and single-point intermittency to fluctuations in the level of scintillation.

### Footnotes

- \*This work was sponsored by the Advanced Research Projects Agency.
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### Captions to Figures

1. Probability distributions for single-point temperature fluctuations.
2. Probability distribution for logarithm of temperature fluctuations.
3. Short-term refractive index structure constant ( $C_n^2$ ) and log amplitude variance of scintillations ( $\sigma^2$ ) vs. time.
4. Fluctuations in spectral components of temperature fluctuations for a 1 Hz bandwidth centered at 2 Hz and 100 Hz, vs. time.

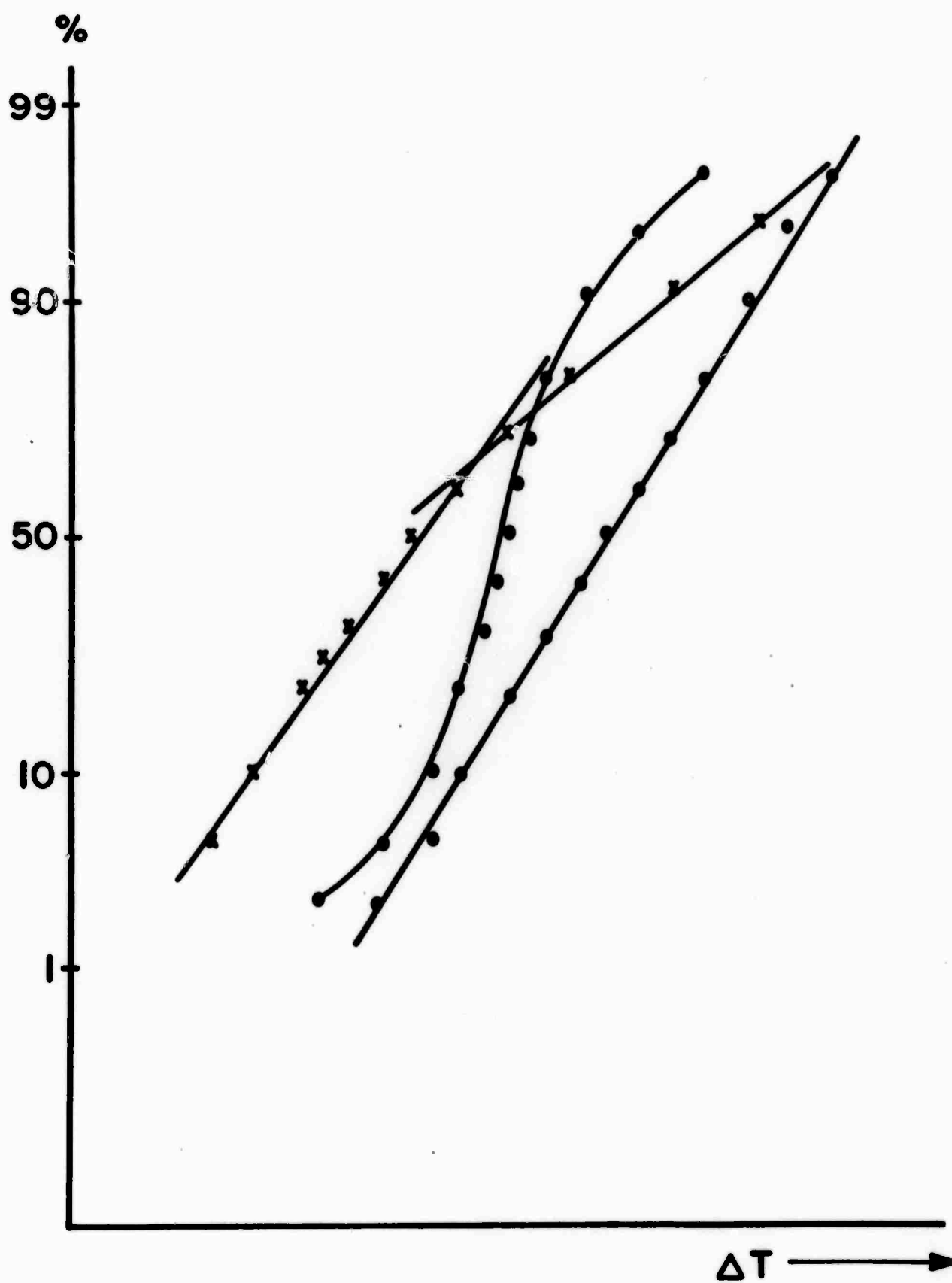


FIGURE 1



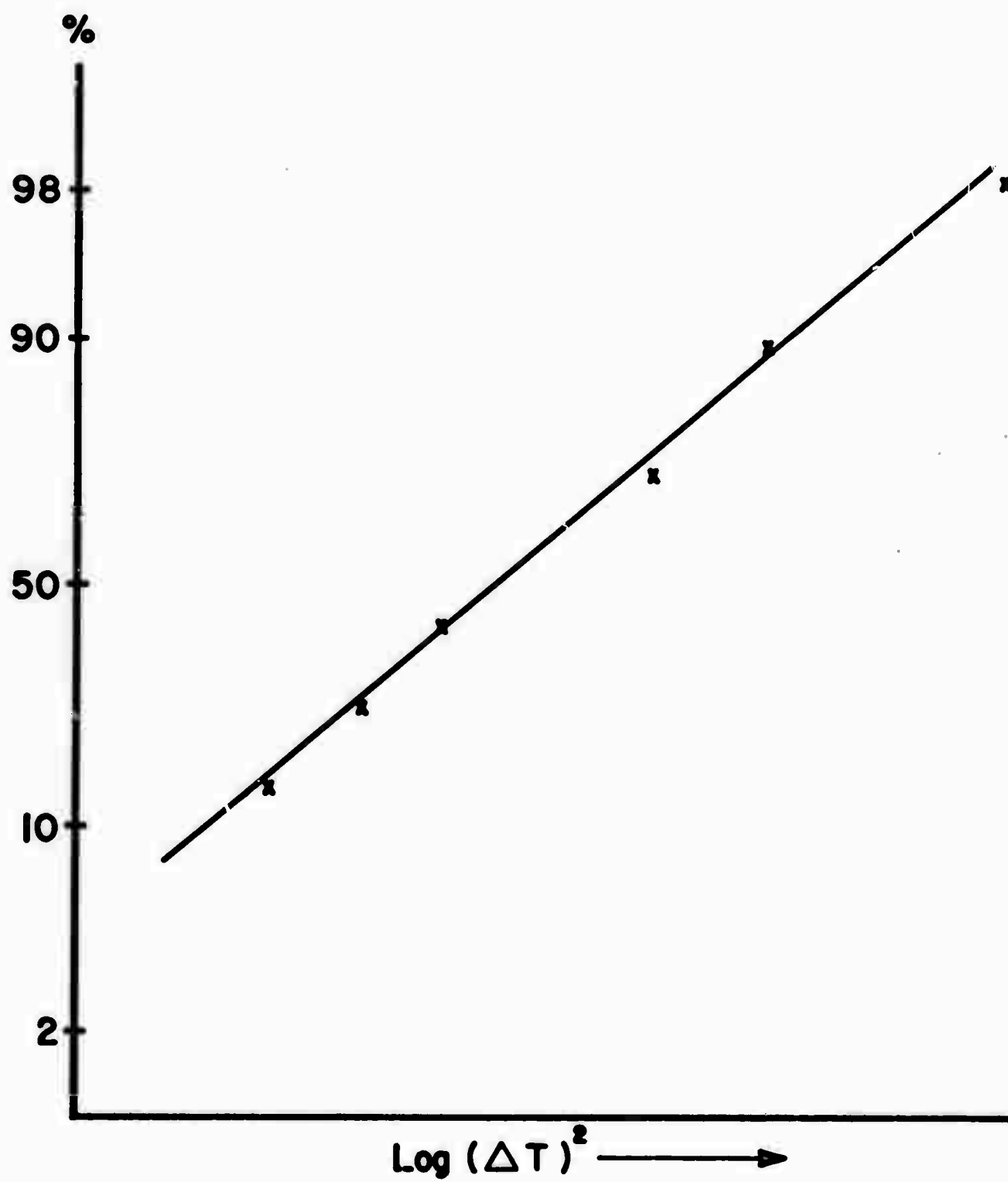


FIGURE 2

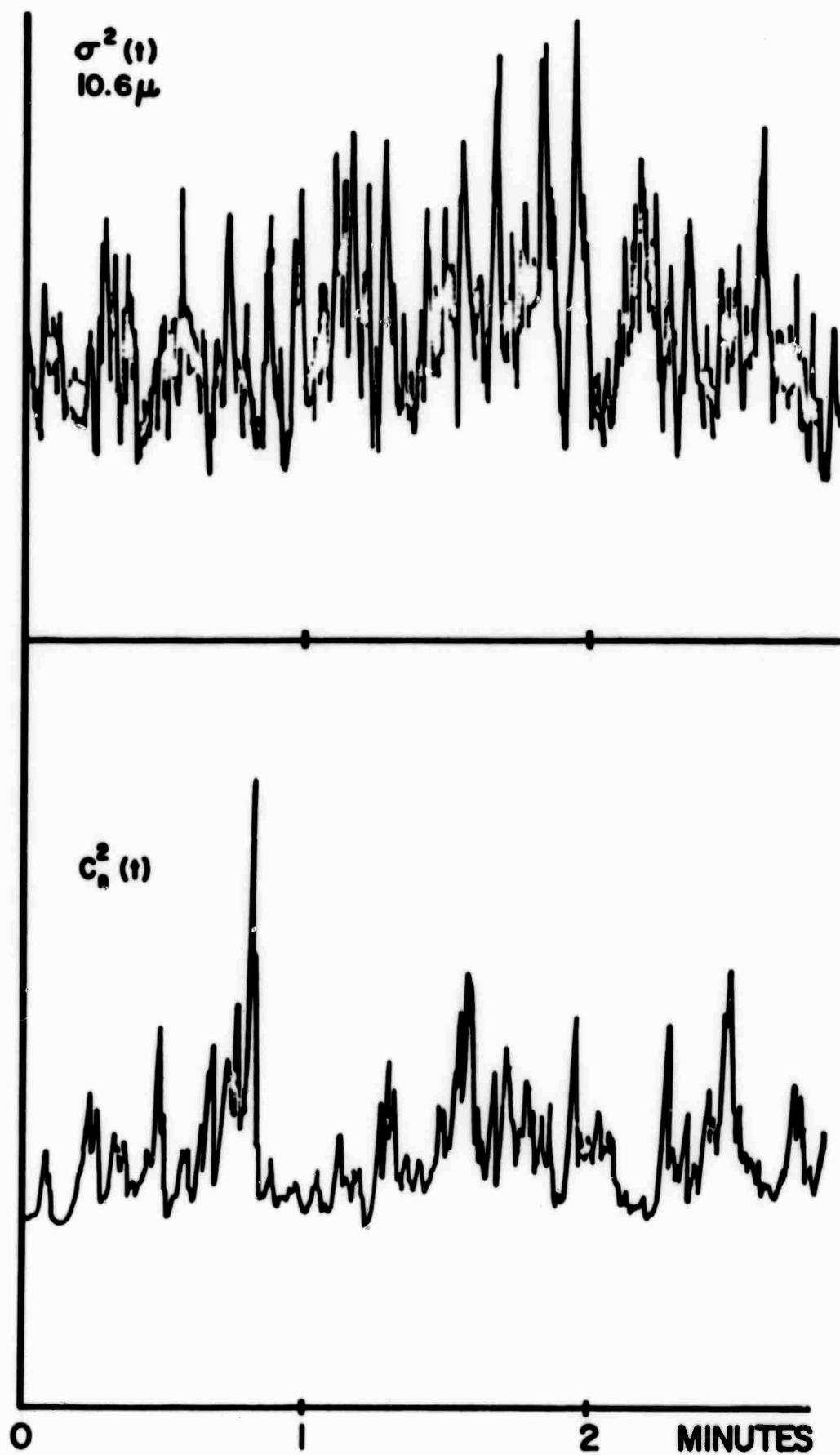


FIGURE 3

B-13

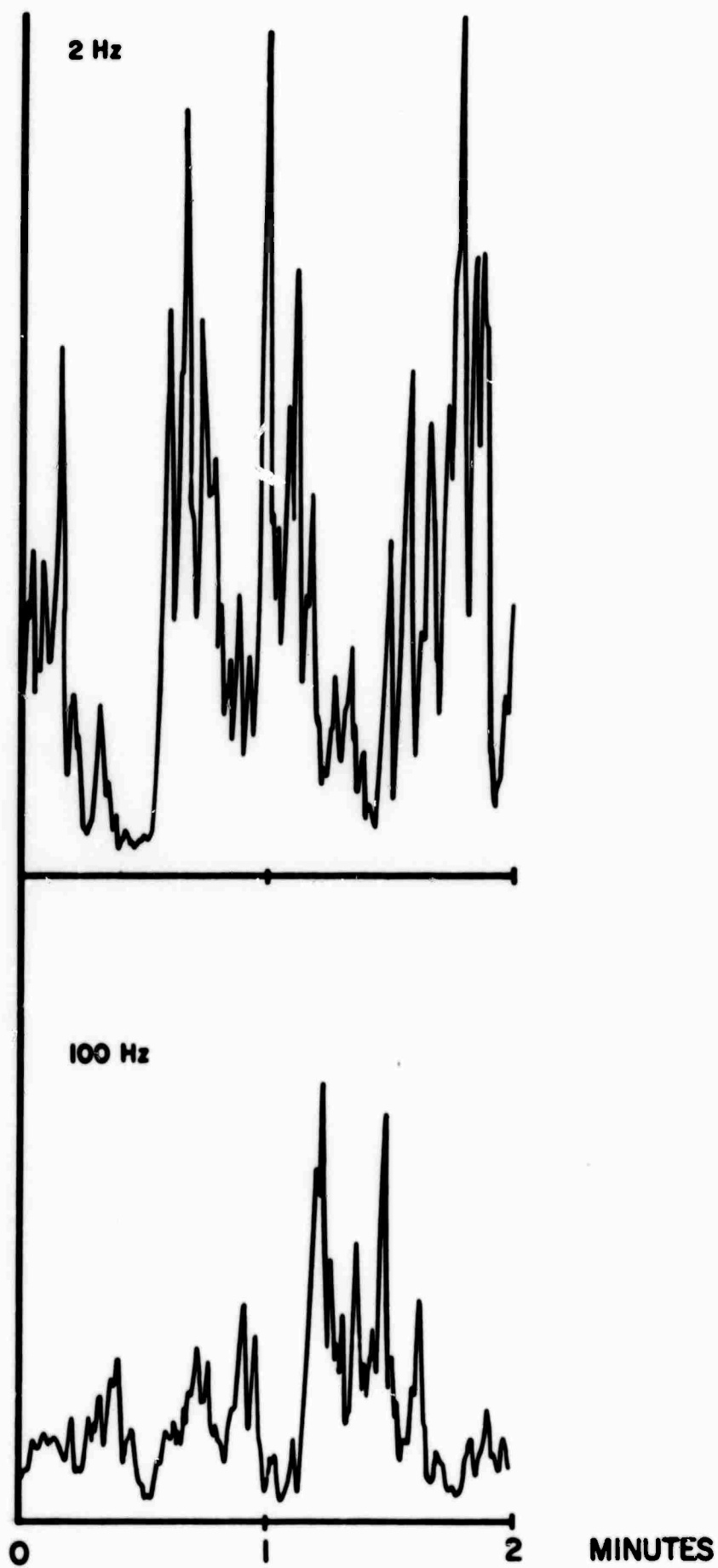


FIGURE 4

B-14

Table I. Representative Results for Turbulence and Scintillation Interdiffusivity Measurements.

Run No.	Transverse Wind Speed (mph)	$\sigma_t^2(4880\text{\AA})$	NORMALIZED VARIANCES					
			Averaging Time (Sec.)	$C_n^2(t)$	$\sigma^2(t)$ 4880\text{\AA}	$\sigma^2(t)$ 10.6	$\Delta T(t)$ 2 Hz	$\Delta T(t)$ 100 Hz
1	8.5	0.066	1	—	0.039	0.017	—	—
			10	—	0.026	0.011	—	—
2	0-4.5	3.0 (saturated)	1	0.25	$5.6 \times 10^{-3}$	0.014	0.81	0.22
			10	0.022	$1.4 \times 10^{-3}$	$1.7 \times 10^{-3}$	—	—
3	≈ 0	5.9 (saturated)	1	1.8	0.021	0.049	0.94	0.23
			10	0.5	$2.5 \times 10^{-3}$	$8.6 \times 10^{-3}$	—	—

## APPENDIX C

### Transmitter Size and Focus Effects on Scintillations\*

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Analyses of turbulence effects on finite laser beams<sup>1-5</sup> have predicted a drastic reduction in scintillation levels when the target or receiver is located in the near field of a focused transmitter. Although this effect has not been well verified, it is being increasingly utilized in system design studies. These analyses of "transmitter aperture averaging" do not apply to saturated scintillations (multiple scattering),<sup>6</sup> but are pertinent to most or all  $10.6\mu$  systems and to vertical paths.<sup>7</sup>

In general, the theoretical results have not been presented in the form of detailed numerical predictions, which are necessary in order to examine such factors as 1) the criticalness of the focus adjustment, 2) the effects of deliberate divergence to alleviate beam wander and/or tracking problems, and 3) the degree to which finite-beam effects may have affected scintillation experiments in the literature. In this note we present generalized curves for predicted scintillations as a function of transmitter size and divergence, and discuss the

implications.

An expression for the on-axis log amplitude variance ( $\sigma^2$ ) for a horizontal laser beam may be obtained from Ref. 3, Eq. 28 as

$$\sigma^2 = 2.18 k^{7/6} L^{11/6} C_n^2 \int_0^1 f(x) dx \quad (1)$$

where

$$f(x) = \operatorname{Re} \left\{ \left( \frac{\alpha_1 L(1-x) + 1 [1 - \alpha_1 L + \alpha_1^2 L^2 x + \alpha_2^2 L^2 x - \alpha_2 L x]}{(1 - \alpha_2 L)^2 + (\alpha_2 L)^2} \right) (1 - x) \right\}^{5/6} \\ - \left[ (1 - x)^2 \left( \frac{\alpha_1 L}{(1 - \alpha_2 L)^2 + (\alpha_2 L)^2} \right) \right]^{5/6} \quad (2)$$

and  $k$  is the optical wavenumber,  $L$  is the path length, and  $C_n^2$  is the refractive index structure constant.<sup>6</sup> The inverse transmitter-Fresnel-number is given by  $\alpha_1 L$ , and  $\alpha_2 L = L/R$  where  $R$  is the radius of curvature of the outgoing wavefront. We have assumed a Kolmogorov turbulence spectrum with a zero inner scale.<sup>4</sup> The above expression is also derived in Ref. 5 using a different technique.

We now define  $\sigma_s^2$  as the variance for the case of a spherical wave or point source:<sup>6</sup>

$$\sigma_s^2 = 0.124 k^{7/6} L^{11/6} C_n^2 \quad (3)$$

We then wish to examine  $(\sigma^2/\sigma_s^2)$  as a function of transmitter size ( $\alpha_1 L$ ) and divergence ( $\alpha_2 L$ ), using computerized numerical integration. Note that collimated and focused conditions correspond to  $\alpha_2 L = 0, 1$  respectively.

Also, for a divergent beam,  $\alpha_2 L$  is related to the receiver and transmitter beam diameters ( $D_{R,T}$ ) and beam divergence angle  $\theta$  by

$$\begin{aligned} -\alpha_2 L &= D_R/D_T \\ \theta &= -\alpha_2 L (D_T/L) \end{aligned} \quad (4)$$

The results are given in Figures 1-3, and suggest several important points. From Figure 1, we note that as the transmitter becomes larger, the criticalness of the focus adjustment increases accordingly, and the theory predicts that the effects of a small misadjustment become very serious--sufficient to increase the variance by orders of magnitude over that for a true focus or even a point source. If true, the mechanism of such scintillation enhancement is not obvious, and the practicality of maintaining the required adjustment (discussed below) is questionable. Note that the details of scintillation reduction with precise focusing would require a logarithmic scale; results vs. transmitter size and inner scale of turbulence are given in Refs. 1-4.

The use of deliberate divergence destroys the "aperture averaging" effect. For the case of turbulence propagation experiments per se, we see from Figure 2 that ( $\alpha_2 L \gtrsim 10^2$ ) ensures point-source results, but a 30% error may result if  $\alpha_2 L = 10$ . From Figures 2 and 3, the use of a large, highly-divergent transmitter ensures results which approach the point-source case; however, a 2% error may result at  $\alpha_2 L = 5$  even for a

fairly small  $D_T$ . Finally, the use of a converging beam which focuses in front of the receiver ( $\alpha_2 L > 1$ ) results in some reduction over the point source case.

The criticalness of focus may be further examined by defining

$$\alpha_2 L - 1 = \Delta \quad (\ll 1) \quad . \quad (5)$$

It may easily be shown that the geometrical focal planes at the transmitter telescope input and receiver region have been respectively moved (relative to the ideal  $\Delta = 0$  case) by

$$\delta = \frac{f^2}{L} \Delta \quad , \quad (6a)$$

$$L - R = L \Delta, \quad (6b)$$

where  $f$  is the focal length of the transmitter telescope. Furthermore, the scintillation-reduction effect will disappear when the geometrical, defocused receiver spot size ( $D_T \Delta$ ) is comparable to the diffraction limit, i.e.

$$\Delta \approx \alpha_1 L \quad . \quad (?)$$

Combining (6a) with (?), we require

$$\frac{\delta}{f} \ll \alpha_1 L \frac{f}{L} \quad . \quad (8)$$

The required transmitter focal plane precision (8) may be a few microns. It is furthermore implied that spatial filtering and



good output optics may be required to achieve the desired reduction.

One difficulty with the above theory is that it apparently does not properly account for beam wander effects. Wander is being studied experimentally and analytically,<sup>8-10</sup> and at least one effort is underway to theoretically combine the phenomena of wander and scintillations.<sup>11</sup> It is usually possible to spectrally separate fading due to wander and scintillations, due to the slow ( $\approx 1$  Hz) variations of the former. However, the presense of beam wander implies that the target will not remain at the short-term beam centroid, with deleterious effects on scintillation which have not yet been numerically evaluated.<sup>3</sup> In addition, turbulence-induced beam spreading is well known to be much greater than the diffraction limit of a reasonably large aperture.<sup>9,10</sup> This may indicate a more fundamental deficiency in the theory.

The theoretical prediction of scintillation reduction through focusing has been questioned in a recent review paper,<sup>6</sup> using an argument related to turbulence-induced wavefront distortion. The condition for the propagation of a meaningful "focused" wave is given as

$$C_n^2 \leq (\text{const.}) \times \frac{(D_T)^{7/3}}{L^3} \quad . \quad (9)$$

However, by comparison with Eq. (3), this condition may be restated as

$$\begin{aligned} \sigma_s^2 &\leq (\text{const.}) \times \left( \frac{D_T^2}{\lambda L} \right)^{7/6} \\ &= (\text{const.}) \times (\alpha_1 L)^{-7/6} \quad , \quad (10) \end{aligned}$$

where the final constant is on the order of 0.02. This condition is largely useless, since it is a weak condition precisely when  $(\alpha_1 L \ll 1)$ , i.e. when the focusing concept itself is applicable.

The above condition was derived<sup>6</sup> through the comparison of a wavefront distortion limitation at the transmitter with a (plane wave) distortion expected at the receiver. However, we have modified this argument to apply at all points of the path, and simply obtain a different  $(-5/6)$  exponent. It seems therefore that first-order theory cannot be expected to predict its own limitations.

It may be mentioned that for a monotonically decreasing turbulence along the path (uplink case), the prediction<sup>7</sup> is that a collimated beam will scintillate similarly to a focused beam in the horizontal case. One should examine<sup>12</sup> the criticalness of the collimation adjustment, and the sensitivity of the scintillation reduction to the actual turbulence profile vs. altitude.

The above considerations were motivated by experiments in which we have not been successful in demonstrating a drastic scintillation-reduction through the use of a large, focused transmitter. We are now modifying the equipment in order to precisely control the focal plane and to attempt to verify the  $(\alpha_1 L = 10^{-1})$  curve in Figure 1a, including both scintillation reduction and enhancement over that for a point source. Experiments are also needed under multiple scattering conditions, and especially for vertical links.

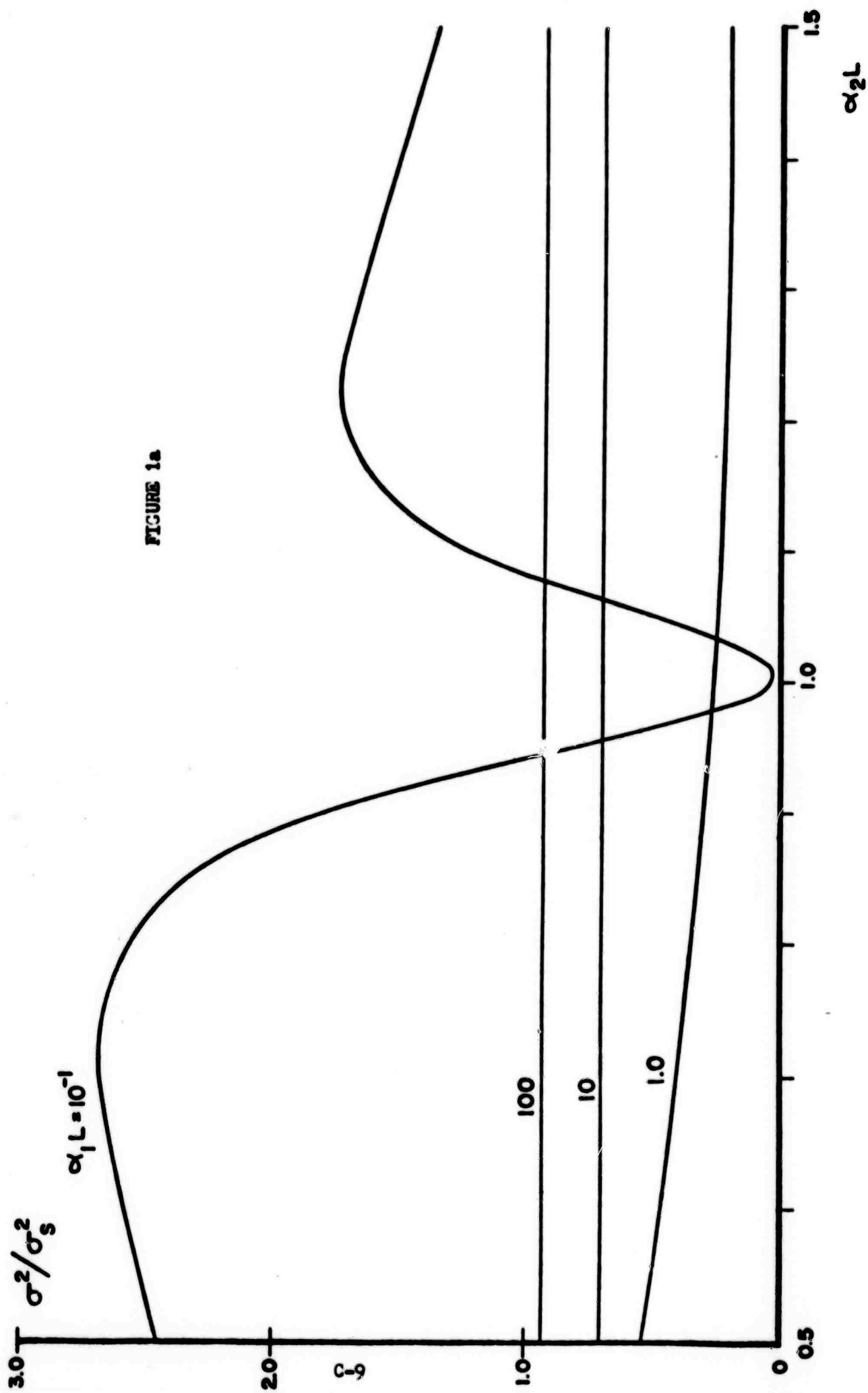
#### Footnotes

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### Captions to Figures

1. Normalized log amplitude variance vs. transmitter focus conditions.
  - a. Small and moderate apertures.
  - b. Large apertures.
2. Normalized log amplitude variance vs. transmitter divergence conditions.
3. Normalized log amplitude variance vs. transmitter inverse Fresnel number.



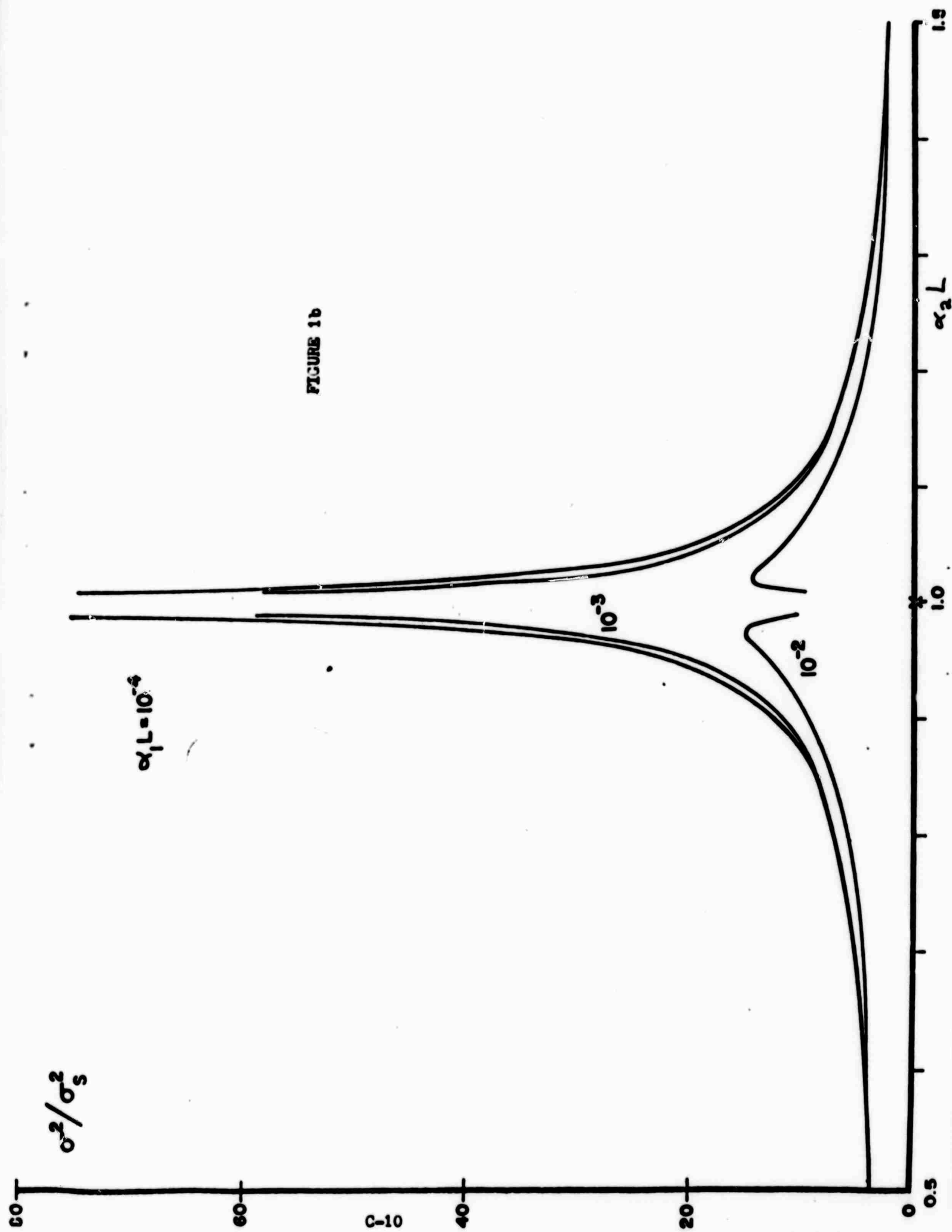


FIGURE 1b

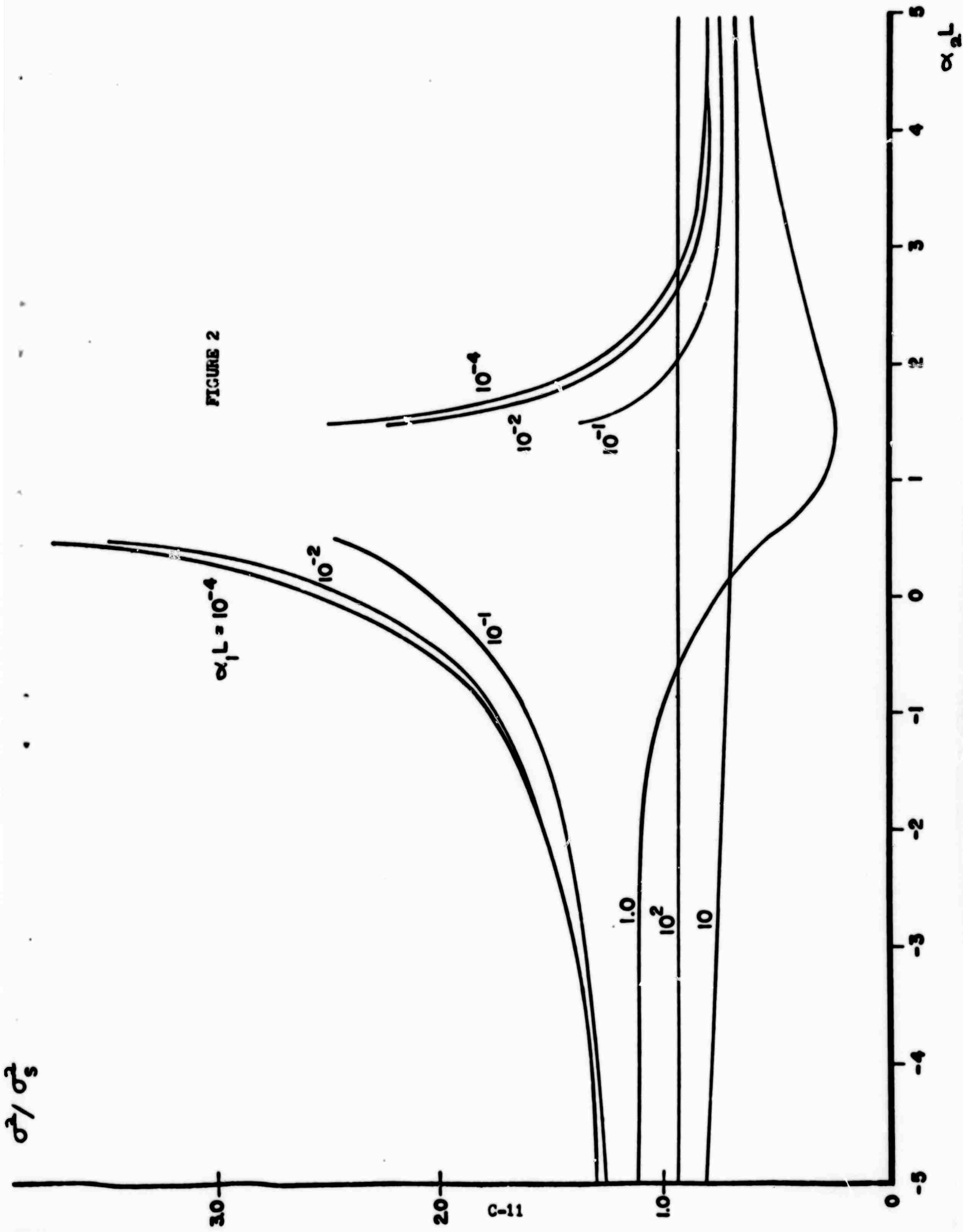


FIGURE 2

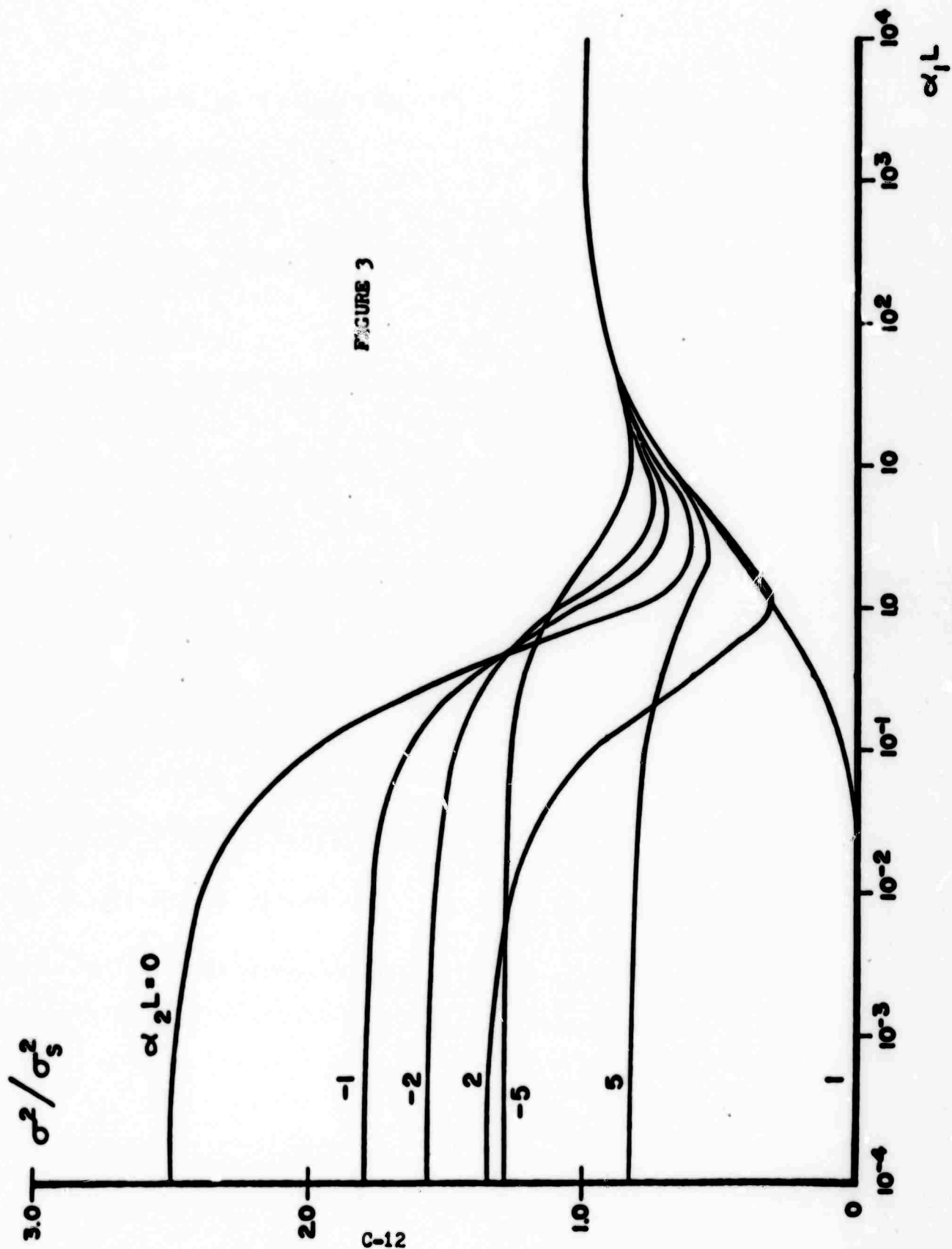


FIGURE 3

C-12